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**Torsional Instabilities  
in Composite  
and Composite-Reinforced  
Aluminum-Alloy  
Thin-Walled Cylinders**

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DEPARTMENT  
OF  
COMMERCE

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# Torsional Instabilities in Composite and Composite-Reinforced Aluminum-Alloy Thin-Walled Cylinders

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# CONTENTS

	Page
1. INTRODUCTION . . . . .	1
2. NOMENCLATURE . . . . .	2
2.1 Units . . . . .	2
2.2 Symbols . . . . .	3
2.3 Notation . . . . .	6
3. ANALYSIS . . . . .	8
3.1 Elastic Properties of a Laminated Tube. . . . .	8
3.2 Torsion Buckling. . . . .	11
4. TORSION SPECIMENS. . . . .	12
4.1 Aluminum-Alloy Torsion Specimen . . . . .	12
4.2 Prepreg Materials . . . . .	13
4.3 Specimen Designations . . . . .	13
5. TORSION SPECIMEN PREPARATION . . . . .	14
5.1 Aluminum-Alloy Tubes. . . . .	14
5.2 Glass/Epoxy Composite Tubes . . . . .	14
5.3 Boron/Epoxy Composite Tubes . . . . .	15
5.4 Glass/Epoxy- and Boron/Epoxy-Reinforced Aluminum-Alloy Tubes. . . . .	16
5.5 Volume Fraction Measurements. . . . .	16
5.6 Profile Measurements. . . . .	17
6. MATERIAL PROPERTY SPECIMENS AND RESULTS. . . . .	17
6.1 Material Property Specimens . . . . .	17
6.2 Results . . . . .	18
7. TORSION TEST PROCEDURE . . . . .	18
7.1 Instrumentation of Test Specimens . . . . .	18
7.2 End Fixtures. . . . .	19
7.3 Testing Machine . . . . .	19
7.4 Testing Procedure . . . . .	19
8. RESULTS FROM TORSION TESTS . . . . .	20
8.1 Elastic Behavior of Specimens . . . . .	20
8.2 Torque-Strain Curves. . . . .	20
8.3 Buckling Strength . . . . .	20
8.4 Buckle Angle and Buckle Length. . . . .	21
9. DISCUSSION . . . . .	21
9.1 Elastic Torsion Behavior. . . . .	21
9.2 Computed $T$ - $\gamma$ Curves . . . . .	21
9.3 Buckling Behavior . . . . .	22
9.4 Post-Buckling Behavior. . . . .	23

CONTENTS-Continued

	Page
10. CONCLUSIONS . . . . .	24
11. REFERENCES . . . . .	25
APPENDIX A MATERIAL PROPERTY TESTS . . . . .	66

# TORSIONAL INSTABILITIES IN COMPOSITE AND COMPOSITE-REINFORCED ALUMINUM-ALLOY THIN-WALLED CYLINDERS

Donald E. Marlowe

The elastic buckling strength has been determined for thin-walled aluminum-alloy tubes fabricated with and without unidirectional boron/epoxy and glass/epoxy composite materials applied as reinforcement to their outer surfaces. Three boron/epoxy ply orientations were investigated. The results of these tests have been compared with the buckling strengths predicted by two analytical techniques. It was found that reinforcement of the metal tubes with an equal thickness of the composite material increased the buckling strength about threefold. The effect on buckling strength of the ply angles investigated is small when compared with the overall effect of adding the reinforcement. The analyses have approximately the same degree of accuracy as that generally attributed to Donnell's treatment of isotropic tubes.

Key words: Aircraft structures; boron/epoxy; composite materials; reinforced aluminum; stability; thin shells; torsion buckling.

## 1. INTRODUCTION

The problems associated with increasing the strength or stiffness of aircraft structural components without increasing the weight of material significantly have long concerned the design engineer. In recent years the development of composite materials, such as boron or graphite fiber-reinforced epoxy, or metal-matrix composite material, has given the designer additional latitude in the solution of these problems. The use of components fabricated entirely from these materials requires an extensive revision of conventional aircraft fabrication techniques. One design concept involves the selective reinforcement of the metallic part with filament-reinforced composite materials instead of complete replacement of metallic components with composite components. Some advantages of this concept are large weight savings with minimal material useage, retention of current joining and fastening technology, and maximum joining reliability and minimum risk along with significant cost-effective weight savings. Some experimental investigations of the benefits to be expected from using composite materials in this way have been made (refs. 1, 2).

The primary purpose of the study reported herein was to investigate the elastic buckling strength of a boron/epoxy-reinforced aluminum-alloy tubular component loaded in torsion. The strength was demonstrated by the testing of tubes fabricated with and without reinforcement. Specimens used for this study consisted of 6061-T6 aluminum-alloy tubes reinforced on the external surface with boron/epoxy, all-boron/epoxy tubes, aluminum-alloy tubes reinforced with glass/epoxy, all-glass/epoxy tubes, and all-aluminum-alloy tubes. The aluminum-alloy tubes and the all-composite tubes were designed with a single diameter to thickness ( $D/t$ ) ratio and in two lengths such that the length to diameter ( $L/D$ ) ratios are one order of magnitude apart.

This investigation was conducted at the National Bureau of Standards under the sponsorship and with the financial assistance of the National Aeronautics and Space Administration, Langley Research Center.

## 2. NOMENCLATURE

### 2.1 Units

The units used for physical quantities defined in this paper are given in both the U.S. Customary Units and in the International System of Units (SI). Conversion factors relating the two systems are given in reference 3, and those pertinent to the present investigation are presented in the Appendix A3 of reference 3.

## 2.2 Symbols

- $A_{ij}$ ,  $i, j = 1, 2, \dots, 6$ . Component of the in-plane stiffness matrix [A] for a laminate.  $A_{ij} = A_{ji}$ . Defined by eq (5).
- $B_{ij}$ ,  $i, j = 1, 2, \dots, 6$ . Component of the stiffness coupling matrix [B] for a laminate.  $B_{ij} = B_{ji}$ . Defined by eq (5).
- D Midsurface diameter of tube.
- $D_{ij}$ ,  $i, j = 1, 2, \dots, 6$ . Component of the flexural stiffness matrix [D] for a laminate.  $D_{ij} = D_{ji}$ . Defined by eq (5).
- $E_{11}$  Young's modulus in the direction of the major in-plane axis of material orthotropy.
- $E_{22}$  Young's modulus in the direction of the minor in-plane axis of material orthotropy.
- $G_{12}$  In-plane shear modulus, referred to the principal axes of material orthotropy.
- H Defined by eq (9).
- h Thickness of laminate.
- $K_{ij}$ ,  $i, j = x, y$ . Component of the plate curvature matrix [K] for a laminate.  $K_{ii} \equiv K_i$ .  $K_{ij} = K_{ji}$ .
- L Length of tube.
- $M_{ij}$ ,  $i, j = x, y$ . Component of the moment resultant vector [M] for a laminate.  $M_{ii} \equiv M_i$ .  $M_{ij} = M_{ji}$ . Defined by eq (3).
- $N_{ij}$ ,  $i, j = x, y$ . Component of the stress resultant vector [N] for a laminate.  $N_{ii} \equiv N_i$ .  $N_{ij} = N_{ji}$ . Defined by eq (3).
- n Number of laminas in a laminate.
- $Q_{ij}$ ,  $i, j = 1, 2, \dots, 6$ . Component of the in-plane stiffness matrix [Q] for a lamina.  $Q_{ij} = Q_{ji}$ . Defined by eq (1).
- $\bar{Q}_{ij}$ ,  $i, j = 1, 2, \dots, 6$ . Component of the in-plane stiffness matrix [Q] for a lamina, transformed to the plane orthogonal coordinate system xy.  $\bar{Q}_{ij} = \bar{Q}_{ji}$ . Defined by eq (2).
- R Midsurface radius of tube.
- T Torque
- t Thickness

$t_c$  Nominal thickness of a composite lamina.

$t_k$  Actual thickness of the kth lamina.

$u, v, w$  Displacements in the  $x, y, z$  directions, respectively.

$x, y, z$  Orthogonal coordinates, generally in the directions of principal axes of loading.

$\gamma_{ij} \equiv \epsilon_{ij}, i \neq j.$

$\epsilon_{ij}, i, j = 1, 2$  or  $i, j = x, y.$  Component of the strain matrix  $[\epsilon]$  in the plane orthogonal coordinate system  $ij.$   $\epsilon_{ii} \equiv \epsilon_i.$   
 $\epsilon_{ij} = \epsilon_{ji}.$

$\theta$  Angle between plane orthogonal coordinate systems  $xy$  and  $12.$

$\nu_{12}$  Major in-plane Poisson's ratio.

$\nu_{21} \equiv \nu_{12} \frac{E_{22}}{E_{11}}$  Minor in-plane Poisson's ratio.

$E_I \equiv E_{11}$

$E_{II} \equiv E_{22}$

$E_{III} \equiv G_{12}$

$E_{IV} \equiv \nu_{12}$

$\sigma_{ij}, i, j = 1, 2$  or  $i, j = x, y.$  Component of the stress vector  $[\sigma]$  in the plane orthogonal coordinate system  $ij.$   $\sigma_{ii} \equiv \sigma_i.$   
 $\sigma_{ij} = \sigma_{ji}.$

$\tau_{ij} \equiv \sigma_{ij}, i \neq j.$

$\psi \equiv 1 - \nu_{12} \nu_{21}$

#### Subscripts

$a$  Aluminum alloy.

$avg$  Average.

$c$  Boron/epoxy composite material.

$cr$  Critical.

$eff$  Effective.

$i, j$  Coordinates in the general plane orthogonal coordinate system  $ij$ .

$k = 1, 2, \dots, n$ . Numerical sequence of a specific lamina in a laminate.

$m$  Matrix material of boron/epoxy composite.

$s = I, II, III, IV$ .

$x, y$  Coordinates in the plane orthogonal coordinate system  $xy$ .

$\alpha = a, c, m$  Material.

$1, 2$  Coordinates in the plane orthogonal coordinate system  $12$ .

### Superscripts

$o$  Midsurface.

$*$  Reduced.

## 2.3 Notation

### a. Laminate Elasticity

The Hooke's Law relationship for a single ply or lamina in the principal material coordinate system is given by

$$\sigma_1 = Q_{11} \varepsilon_1 + Q_{12} \varepsilon_2$$

$$\sigma_2 = Q_{12} \varepsilon_1 + Q_{22} \varepsilon_2$$

$$\tau_{12} = Q_{66} \gamma_{12}$$

or

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

or

$$[\sigma] = [Q] [\varepsilon]$$

where

$$Q_{11} = \frac{E_{11}}{\Psi}, \quad Q_{22} = \frac{E_{22}}{\Psi}, \quad Q_{12} = \frac{\gamma_{12} E_{22}}{\Psi},$$

$$Q_{66} = G_{12}, \quad Q_{16} = Q_{26} = 0 \tag{1}$$

and

$$\Psi = 1 - \gamma_{12} \gamma_{21}, \quad \gamma_{21} = \gamma_{12} \frac{E_{22}}{E_{11}}.$$

The Hooke's Law relationship for an arbitrary rectangular coordinate system  $xy$  lying in the plane of the lamina and making an angle  $\theta$  with the principal material coordinate axes is given by (ref. 4):

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

or

$$[\sigma] = [\bar{Q}] [\varepsilon]$$

where

$$\begin{aligned}
 \bar{Q}_{11} &= Q_{11} \cos^4 \theta + 2(Q_{12} + 2 Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \\
 \bar{Q}_{22} &= Q_{11} \sin^4 \theta + 2(Q_{12} + 2 Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\
 \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4 Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12} (\sin^4 \theta + \cos^4 \theta) \\
 \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2 Q_{12} - 2 Q_{66}) \sin^2 \theta \cos^2 \theta + \\
 &\quad Q_{66} (\sin^4 \theta + \cos^4 \theta) \\
 \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2 Q_{66}) \sin \theta \cos^3 \theta + \\
 &\quad (Q_{12} - Q_{22} + 2 Q_{66}) \sin^3 \theta \cos \theta \\
 \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2 Q_{66}) \sin^3 \theta \cos \theta + \\
 &\quad (Q_{12} - Q_{22} + 2 Q_{66}) \sin \theta \cos^3 \theta
 \end{aligned} \tag{2}$$

The elastic behavior of a laminate or layup of plies is determined by suitably combining the behaviors of the individual laminas. Consider a rectangular coordinate system  $x, y, z$  where the origin of the system is in the midplane of the laminate and  $z$  is normal to the plane. The individual laminas are numbered  $1, 2, \dots, k, \dots, n$  beginning from the most negative value of  $z$ . Thus the coordinates which determine the thickness of the  $k$ th lamina are  $z_k$  and  $z_{k+1}$ . The stress and moment resultants on the laminate are

$$\begin{aligned}
 [N] &= \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k dz \\
 [M] &= \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k z dz.
 \end{aligned} \tag{3}$$

Let the midsurface strains in the laminate be represented by

$$[\varepsilon^{\circ}] = \begin{bmatrix} \varepsilon_x^{\circ} \\ \varepsilon_y^{\circ} \\ \gamma_{xy}^{\circ} \end{bmatrix}$$

and the plate curvatures by

$$[K] = \begin{bmatrix} K_x \\ K_y \\ K_{xy} \end{bmatrix}$$

Then, for small deflections, the constitutive equations for the laminate, reference 4, are

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ} \\ K \end{bmatrix} \quad (4)$$

where

$$\begin{aligned} A_{ij} &= \sum_{k=1}^n (\bar{Q}_{ij})_k (z_{k+1} - z_k) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^n (\bar{Q}_{ij})_k (z_{k+1}^2 - z_k^2) \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^n (\bar{Q}_{ij})_k (z_{k+1}^3 - z_k^3) \end{aligned} \quad (5)$$

### 3. ANALYSIS

#### 3.1 Elastic Properties of a Laminated Tube

##### a. Elastic Behavior

Consider a laminated, thin-walled, circular tube. The origin of the coordinate system  $x, y, z$  is taken at one end of the tube at the mid-thickness. Coordinate  $x$  is axial,  $y$  is circumferential and  $z$  is radial, positive outward. The tube is subjected to axisymmetric torsional loading through clamped ends such that one end is free to

rotate and to extend or contract. The boundary conditions for this case, reference 5, are

$$u(0, y) = v(0, y) = w(0, y) = \frac{\partial w(0, y)}{\partial x} = 0$$

$$w(L, y) = \frac{\partial w(L, y)}{\partial x} = 0$$

$$u(L, y) = \text{constant}$$

$$v(L, y) = \text{constant}$$

It is assumed that

$$N_x = 0 \quad \text{and} \quad N_{xy} = \text{constant.}$$

Then, following the analysis of reference 5, if it is further assumed that

$$K_x = K_y = K_{xy} = 0$$

then it may be shown that

$$N_y = 0,$$

that  $\epsilon_x^0$ ,  $\epsilon_y^0$  and  $\gamma_{xy}^0$  are constant, denoting a uniform strain distribution in the midsurface, and that

$$u^0 = \epsilon_x^0 x, \quad v_{xy}^0 = \gamma_{xy}^0 x, \quad w = \epsilon_y^0 R.$$

The expression for the radial displacement  $w$  does not satisfy the boundary conditions but it is claimed (ref. 5) that this is of little consequence to results obtained at short distances from the constrained end.

Under these conditions the constitutive equation, eq (4) reduces to

$$\begin{bmatrix} 0 \\ 0 \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$$

Letting the effective torsional modulus be

$$G_{\text{eff}} = \frac{(\tau_{xy})_{\text{avg}}}{\gamma_{xy}^0} = \frac{N_{xy}}{h\gamma_{xy}^0}$$

it is easily shown that

$$G_{\text{eff}} = \frac{A_{66} + A_{11} A_{26}^2 - 2A_{12} A_{16} A_{26} + A_{22} A_{16}^2}{A_{12}^2 - A_{11} A_{22}} \quad (6)$$

and that the extension-shear coupling factors are

$$\frac{\epsilon_x^0}{\gamma_{xy}^0} = \frac{A_{12} A_{26} - A_{22} A_{16}}{A_{11} A_{22} - A_{12}^2}$$

$$\frac{\epsilon_y^0}{\gamma_{xy}^0} = \frac{A_{12} A_{16} - A_{11} A_{26}}{A_{11} A_{22} - A_{12}^2} \quad (7)$$

#### b. Rule of Mixtures

For the  $k$ th ply or lamina of a composite laminate, a correction to the elastic constants must be made where the actual thickness of the lamina,  $t_k$ , differs from the nominal thickness,  $t_c$ . In this report a simple rule-of-mixture correction is used, that is

$$(\epsilon_s)_k = \frac{t_c (E_s)_c + (t_k - t_c) (E_s)_m}{t_k} \quad (8)$$

If a laminate is cured such that the thickness of each composite lamina exceeds the nominal thickness, then each lamina can be visualized as an effective laminate consisting of one reinforced ply having the nominal thickness plus one ply of unreinforced matrix material where the overall thickness of the combination equals the actual thickness of the lamina. The values of  $A_{ij}$ , eq (5), computed for this effective laminate are identical to those<sup>ij</sup> obtained using the

rule-of-mixtures correction. This correction is entirely valid, therefore, insofar as eqs (6) and (7) are concerned. This argument also holds for laminas which have thicknesses which are less than the nominal one.

This approach is considered to be appropriate for the estimation of the actual values of  $E_{11}$  and  $\gamma_{12}$ . The error introduced by this method of estimating the actual values of  $G_{12}$  and  $E_{22}$  was quite small and it will be shown that for these tests, the results were not significantly affected.

### 3.2 Torsion Buckling

#### a. Torsional Buckling of Orthogonal-Anisotropic Cylinders

Hayashi, reference 6, proposed a theory of elastic torsional buckling in which the critical torques are expressed as a function of the extensional and bending stiffnesses of the tube.

From eq (4),

$$[N] = ([A] - [B][D^{-1}][B])\epsilon^0 + [B][D^{-1}][M]$$

Thus, if the extensional stiffness in the axial direction,  $A_{11}^*$ , is defined as the ration  $N_x/\epsilon_x^0$  when  $[M]$  is identically zero, then it may be calculated from

$$[A^*] = [A] - [B][D^{-1}][B].$$

In a similar way the bending stiffnesses are given by

$$[D^*] = [D] - [B][A^{-1}][B]$$

For a tube of moderate length with clamped ends the critical shear stress resultant is (ref. 6)

$$(N_{xy})_{cr} = \frac{12 D_{22}^*}{L^2} [4.6 + \sqrt{7.8 + 1.67H^{3/2}}]$$

where

$$H = \frac{L^2}{2R} \sqrt{\frac{A_{11}^* (A_{11} A_{22} - A_{12}^2)}{12 D_{22}^* A_{11} A_{22}}} \quad (9)$$

and "moderate length" is defined as

$$\left(\frac{L}{2R}\right)^4 < 7.8 H.$$

This theory was derived for specially orthotropic tubes under the assumption that the tangent of the angle between the buckles and the tube axis is small compared with unity. Solutions based on this theory are compared with the experimental results.

#### b. Buckling of Generally Orthotropic Cylinders

The computer program, "Buckling of Generally Orthotropic Cylinders", which was developed by Chao (ref. 7) was used to compare the predicted buckling torques with the experimental results. In the program, buckling loads are found for a multilayered cylindrical shell loaded in combinations of torsion, axial compression and lateral pressure. The theory treats orthotropic layers whose principal material axes can be oriented in any direction. The shell is assumed to be anisotropic with discrete longitudinal and rib stiffeners which are closely spaced. The elastic properties of the stiffeners are then considered to be smearable in the two directions over the shell. The program seeks the solution with the lowest buckling strength by iterating on the number of circumferential buckling waves.

### 4. TORSION SPECIMENS

Specimens tested in this investigation were intended to show the advantages to be gained in the elastic buckling behavior of thin-walled aluminum-alloy tubes by reinforcing the external tube surfaces with boron/epoxy composite materials. This behavior was determined for tubes of two lengths with a single ratio of D/t.

#### 4.1 Aluminum-Alloy Torsion Specimen

The nominal outer diameter of all specimens is 6 in (15 cm). The specimens were tested with gage lengths of 20.0 in (50.8 cm) and 2.00 in (5.08 cm). The all-aluminum-alloy tubes have a nominal wall thickness of 0.022 in (0.56 mm). The results of tests performed on these tubes were compared with results from similar tubes which had been reinforced with glass/epoxy or boron/epoxy prepreg on the aluminum-alloy tube. The thickness of the reinforcement was approximately equal to the thickness of the aluminum-alloy tube.

## 4.2 Prepreg Materials<sup>†</sup>

The boron/epoxy prepreg used for the reinforcement of the aluminum-alloy tubes and for the all-composite torsion specimens was supplied by the AVCO Corporation. The boron filaments were nominally 0.004 in (0.1 mm) in diameter. The filament had a guaranteed average tensile strength of 400,000 lbf/in<sup>2</sup> ( $28 \times 10^8$  N/m<sup>2</sup>) and an average tensile modulus of elasticity of between 55 and 60 x 10<sup>6</sup> lbf/in<sup>2</sup> (380 and 414 x 10<sup>9</sup> N/m<sup>2</sup>). It was preimpregnated with Whittaker Corporation's NARMCO 5505 RIGIDITE epoxy resin system. The prepreg was provided in two forms, 0.125-in (0.318-cm) tape and broadgoods. The filament count for the tape material as reported by the manufacturer was 26 for the 0.125-in (0.318 cm) tape width; and for the broadgood material was 208 per inch of width (82 per cm). The broadgoods material was supplied with a 0.001 in (0.03 mm) type 104 fiberglass scrim cloth to facilitate handling. This material was stored at 0°F (-18° C) prior to use. All specimens were fabricated within seven months after receipt of the boron/epoxy prepreg.

The glass/epoxy prepreg used in the program was 3M Corporation's SCOTCHPLY E-glass prepreg type 1002. This material has a guaranteed tensile strength of 160,000 lbf/in<sup>2</sup> ( $11 \times 10^8$  N/m<sup>2</sup>) and a tensile modulus of 5.7 x 10<sup>6</sup> lbf/in<sup>2</sup> ( $39 \times 10^9$  N/m<sup>2</sup>). It was provided in 0.5 in (1.3 cm) and 6.0 in (15 cm) wide tapes. This material was also stored at 0 °F (-18° C) prior to use.

## 4.3 Specimen Designations

The system for designating specimens differentiates between materials and specimen length, as follows:

AL series - all-aluminum alloy, long length (20 in, 50.8 cm)

AS series - all-aluminum alloy, short length (2 in, 5.08 cm)

BL series - all-boron/epoxy, long length

BS series - all-boron/epoxy, short length

GL series - all-glass/epoxy, long length

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<sup>†</sup>Certain materials are identified in this paper by trade name in order to adequately specify the experimental procedure. In no case does such identification imply recommendation or endorsement by the National Bureau of Standards, nor does it imply that the material identified is necessarily the best available for the purpose.

ABL series - boron/epoxy-reinforced aluminum alloy, long length

ABS series - boron/epoxy-reinforced aluminum alloy, short length

AGL series - glass/epoxy-reinforced aluminum alloy, long length

## 5. TORSION SPECIMEN PREPARATION

### 5.1 Aluminum-Alloy Tubes

All-aluminum-alloy tubes were machined from a single lot of 6-in (15-cm) diameter, 0.12-in (0.32-cm) thick, 6061-T6 aluminum-alloy seamless drawn tubing. The specimens were cut to length, then turned on a lathe to an outer diameter of 5.950 in (15.11 cm). The dimensions of these specimens are given in table 1.

The short tube specimen is shown in figure 1a. The inner surface of the gage length of the specimen was turned on a lathe to an inner diameter of  $5.906 \pm 0.002$  in ( $15.000 \pm 0.005$  cm).

The long tube specimen is shown in figure 1b. The specimen, after the outer surface was machined, was clamped over its entire length in a specially made jig. The inner surface of the tube was then machined to a final diameter of  $5.906 \pm 0.002$  in ( $15.000 \pm 0.005$  cm) in a boring mill.

### 5.2 Glass/Epoxy Composite Tubes

Several glass/epoxy composite tubes were fabricated and tested. A soft wood mandrel covered with a heat-shrinkable fluorocarbon plastic tube was used. The mandrel was laminated from four pieces of redwood. After removal from the press, the wood was allowed to stabilize for 48 hours before machining. Final finish was obtained with 280-grit sand paper. A length of type FEP heat-shrinkable, fluorocarbon tubing was shrunk into place covering the entire surface of the mandrel (fig. 2).

Two techniques were used for laying up the composite tube. For the tubes with a 90-degree ply-orientation angle, the mandrel was gripped in the jaws of a lathe and the prepreg tape wound onto the mandrel (fig. 3). For this purpose, 0.5-in (1.3-cm) tape was used. The rotation of the mandrel was always in the same direction. Succeeding plies were applied in the same manner after advancing the lathe tool holder one-half the width of the tape. This insured that the tape seams in adjacent plies were staggered. Plies were all started from the same end of the tube.

Tubes with a 0-degree ply orientation were laid up using a prepreg tape 6.0 in (15 cm) wide. The seams between the widths of the tape were staggered in adjacent plies.

After the plies were laid up, a length of the fluorocarbon tubing was shrunk over the laminate, and a thermocouple was embedded between the two layers of fluorocarbon. The part was then cured using the prepreg manufacturer's suggested temperature-time cycle.

The dimensions of these tubes are given in table 1.

### 5.3 Boron/Epoxy Composite Tubes

These specimens were fabricated using the techniques described above for the manufacture of glass-epoxy tubes. For the majority of the tubes with a 90-degree ply-orientation angle, boron/epoxy prepreg tape of 0.125 in (0.32 cm) wide was used (fig. 4). This tape was applied to the fluorocarbon-covered mandrel, taking care to insure that the seams between tapes did not coincide in adjacent plies. The resulting ply angle was 89.5 degrees. One short tube with a 90-degree ply-orientation angle, specimen BS-4, was fabricated by rolling a continuous 5-in (13-cm) wide strip of broadgoods material onto the mandrel until four plies had been built up.

Tubes with a 0-degree ply orientation were laid up with a single sheet of boron/epoxy prepreg material per ply (fig. 5). As noted, this material was supplied with a fiberglass scrimcloth carrier. The plies were laid on the mandrel with the scrim cloth on the outside.

Tubes with a 45-degree ply orientation were laid up using strips of boron/epoxy prepreg material. Because of the difficulty of wrapping wide sheets of material at 45 degree angles, the broadgoods were slit into 3-in (8-cm) wide strips (fig. 6). Care was taken to insure that the seams of strips in adjacent layers of prepreg did not coincide.

After four plies of prepreg had been built up, a layer of release cloth was placed over the laminate (fig. 7). A layer of 0.004-in (0.01-cm) fiberglass bleeder cloth was placed over the release cloth and sealed at the seam with mylar tape (fig. 8). A section of fluorocarbon tubing was then shrunk over the laminate (fig. 9), a thermocouple was embedded between the two layers of fluorocarbon and the part was cured using the temperature-time cycle recommended by the prepreg manufacturer. The dimensions of the boron/epoxy specimens are given in table 1.

Specimen BL-2A was salvaged from specimen BL-2 after the latter tube had been tested and had failed locally near one end so that a 15-in (38-cm) specimen could be recovered.

## 5.4 Glass/Epoxy- and Boron/Epoxy-Reinforced Aluminum-Alloy Tubes

The aluminum-alloy tubes which were intended for testing as reinforced tubes were chemically cleaned immediately prior to the layup process. The tubes were vapor degreased in trichlorethylene, washed in a sodium hydroxide solution, deoxidized and rinsed in hot water. They were dried in a forced air jet.

The composite layup techniques for these tubes were the same as those for the all-composite tubes where the aluminum tube served as a non-removable mandrel. Specimens ABL-1, ABL-2 and ABL-3, the first reinforced aluminum tubes fabricated, were cured without the bleeder cloth under the heat-shrinkable tubing. The dimensions of these specimens are shown in table 2.

## 5.5 Volume Fraction Measurements

After all mechanical testing had been completed, a sample of boron/epoxy material was removed from fifteen tubes. A photomicrograph of a cross-section of each sample was used to determine volume fractions of epoxy and boron. These measurements were made using the grid-intersection technique commonly used in sterology (ref. 8). The results of these measurements are shown in table 3. Typical photomicrographs used are shown in figure 10.

As discussed above, during the fabrication of the specimens, three combinations of reinforcement material and fabrication techniques resulted in three groups of specimens based on the volume fraction measurements. Specimens were fabricated from broadgoods material containing fiberglass scrim cloth both with and without the use of a fiberglass bleeder cloth in the curing cycle. Specimens were also wound from narrow tape without scrim cloth but with a fiberglass bleeder cloth. These three layup techniques resulted in tubes with differing ratios of boron filament to epoxy matrix. The tubes fabricated with broadgoods material having a scrim cloth, but without a bleeder cloth, have a boron fiber fraction of approximately 34 percent. Specimens using the same broadgoods material, and also using a bleeder cloth, have a boron fiber fraction of about 43 percent. The tape wound, 90 degree specimens fabricated without scrim cloth, but with a bleeder cloth, have a boron volume fraction of about 55 percent. It should be recalled at this point that the fiberglass scrim cloth constitutes about 10 percent of the thickness of a finished laminate. This alone tends to a somewhat thicker laminate and a lower boron fraction.

## 5.6 Profile Measurements

Surface imperfection measurements were made on seven aluminum-alloy tubes. Five of these were tested with boron/epoxy reinforcement. To make these measurements, the tube was supported at its ends in the jaws of a lathe. A dial gage graduated in 0.0001 in (0.00025 cm) per division was mounted in the tool holder and moved to trace a path along a generator line of the tube. Measurements were taken at 1-in (2.5-cm) intervals along the gage length of the tube (fig. 11) and along five generator lines equally spaced around the circumference of the tube. The results of the measurements are shown in figure 12.

Elevations, or radial displacements from the straight generator line, are measured relative to the elevation at the left end of the No. 1 profile, i.e., relative to the first elevation point measured. For example, the generator profiles for tube AL-3, figure 12a, indicate a maximum radial displacement, from the first point on the No. 1 profile, of 0.007 in (0.018 cm) at a location 15 in (39 cm) from the left end of the tube. It would also be possible, with these measurements, to indicate the amount of out-of-roundness of the tube at any station along its length.

When these tubes were instrumented for test, the strain gage rosettes were centered over the profile lines.

Figure 12 also indicates the profile on which the first indication of gage reversal was observed. Gage reversal is defined as a change in direction of a strain increment during a torque increment experienced by either of the 45-degree gages of a three-gage rosette. Where multiple profiles are indicated, gage reversal occurred simultaneously in more than one rosette. The ABS-series tubes failed catastrophically and an indication of buckling prior to gross failure was generally not available. Correlation between the surface imperfections and the location of the first buckle indication was not evident from this work.

## 6. MATERIAL PROPERTY SPECIMENS AND RESULTS

### 6.1 Material Property Specimens

Material property tests for the determination of longitudinal elastic modulus, transverse elastic modulus and Poisson's ratio were made on all materials used in this program. The preparation of and testing procedure for these specimens are described in the Appendix to this report.

## 6.2 Results

The average material properties measured on the aluminum-alloy, boron/epoxy, and glass/epoxy materials are shown in table 4. These values of  $E_{11}$ ,  $E_{22}$ , and  $\nu_{12}$  were corrected for thickness variation using equation 8 when they were used for calculation with the two analytical methods of buckling analysis described in Section 3 above.

## 7. TORSION TEST PROCEDURE

### 7.1 Instrumentation of Test Specimens

#### a. Strain Gages

Each of the twenty-nine tubular test specimens was gaged at the midlength of the specimen. Five 45-degree rosette foil strain gages having a gage length of 0.25 in (0.63 cm) were equally spaced around the specimen. These rosettes were oriented to measure strains in the axial direction and at 45 degrees to the axial direction. Tubes in the AL, GL, BL and BS series were gaged on the outer surfaces (fig. 13). On the ABL, AGL, ABS and AS series specimens, the gages were mounted on the inner surfaces (fig. 14). Specimen AL-1, the first tube tested, had gages in locations other than those mentioned above. This specimen was instrumented in three belts of three rosette gages each. These belts were located at 0.25, 0.50 and 0.75 of the length (fig. 15). In addition, single gages were mounted near one end to measure the end effects due to the end caps.

In addition to the above gage locations, several specimens were tested with additional strain gages in an attempt to detect the earliest indication of buckling and to check uniformity of strain distribution. Specimens AS-1 and AGL-1 each had five single-element gages mounted on the same belt as the five rosette gages and spaced between them. These gages were oriented at 45 degrees, in the direction of the maximum compressive stress. Several reinforced aluminum-alloy specimens had one rosette gage mounted on the outside of the tube back-to-back with a similar gage on the inner surface.

On tubes where the surface waviness profiles had been measured, the rosette gages were centered on the generators along which the measurements had been made. All gages were mounted using a cynoacrylate contact cement. Each gage was sealed from atmospheric damage with a polyurethane coating.

#### b. Twist Measurements

Twist measurements were made on all specimens with the exception of Specimen AL-1. Measurements of total twist were made at each end of the gage length of the specimen. The difference between the two

measurements determined the net twist on the specimen. These measurements were made using the reflected light from two mirrors bonded to the specimen at the ends of the gage length. The light source and angle sensor was a Tuckerman optical strain gage autocollimator. The autocollimator provided a source of plane, parallel light which was reflected from the mirrors and read manually on the scale of the autocollimator (fig. 16). The smallest angle of twist which could be resolved using this troptometer was one milliradian.

## 7.2 End Fixtures

The end fixtures used for each specimen are shown in figure 17. These close-fitting end plugs were bonded to the inner surface of each end of the tube specimen. There are approximately 27 in (174 cm) of bonded area at each end of the tube. Three epoxy systems were used at different times during the program for this purpose. The first of these is a room temperature-curing epoxy which was used where low torsional loads were expected. The other two epoxy systems required an elevated temperature cure cycle. This was achieved by the use of infrared radiant lamps or close-fitting heater collars in the area to be cured (figs. 18 and 19). The end plugs for tubes in the AL, GL, BL, and BS series were bonded to the tubes with the specimens mounted in the testing machine, after lead wires were attached to the strain gages. All other specimens had one end plug attached before lead wires were attached to the strain gages. The second end plug was attached when the specimen was placed in the testing machine.

## 7.3 Testing Machine

The torsion testing machine which was used for these tests was calibrated with dead weight and a lever arm just prior to the first torsion test.

The load errors of the testing machine over the range of loads used in these tests did not exceed one percent. The alinement of the testing machine was verified during the calibration procedure. It was observed that the axes of rotation of the two machine heads were coincident within 0.001 in (0.003 cm). The maximum axial force introduced into the specimen, resulting from machine friction in the sliding spindle of the loading head, was measured to be 45 lbf (200 N).

## 7.4 Testing Procedure

After a specimen was bonded to its end plugs in the testing machine, the strain gage leads were wired to the strain measurement circuits and the mirrors and autocollimators of the troptometer were alined. Loads were applied at a constant rate in each load increment. Loading was stopped periodically and load was maintained while strain and twist measurements were recorded. The magnitude of the load

increment between successive sets of readings was decided on the basis of the specimen behavior during the previous increment. The direction of loading was generally the same for all specimens. This was chosen so as to cause a tensile stress in the filaments for tubes fabricated with a 45-degree ply-orientation angle. Thus, the specimens with a 90-degree filament orientation were loaded in the direction which "tightened" the wrap of fibers on the tube. The exceptions to this direction are as follows: the specimen pairs ABL-2 and ABL-4 were loaded in opposite directions and specimens AS-1 and AS-2 were loaded in the direction of ABL-2, which was opposite to all other specimens.

## 8. RESULTS FROM TORSION TESTS

### 8.1 Elastic Behavior of Specimens

The elastic properties of the tubes tested are shown in table 5. This table gives the values of  $G_{\text{eff}}$  and  $\frac{\epsilon_x + \epsilon_y^0}{2\gamma_{xy}^0}$ , the average of the extension shear coupling factors, which were predicted by the analysis and measured during tests. The experimental moduli and average coupling factors shown were determined from the data of the first four torque increments applied in the linear elastic range of each test.

### 8.2 Torque-Strain Curves

The torque-strain curves for all tubes tested are shown in figure 20a thru 20i. The shear strains computed from the rosette measurements at the center of the specimen and averaged over the five rosettes, and the strains computed from the twist measurements of the mirror-autocollimator trolometer, are both shown.

### 8.3 Buckling Strength

The maximum torques and the torques at which gage reversal occurred are summarized in table 6. Included in this table are the values of buckling torque predicted by the Chao and Hayashi analyses used and the ratios of torque at strain reversal to predicted torque for the two analytical programs. The elastic material properties used in the analyses to predict the buckling are those presented in table 4. In the case of the Chao program, these were adjusted for thickness variations from the nominal. The values of  $G_{12}$  used in the Chao program are the averages of  $G_{\text{eff}}$  measured on the AL, BL, and GL specimens, respectively, corrected for variations from nominal ply thickness using the rule of mixtures (eq. 8).

Photographs of typical failed specimens are shown in figures 21a through 21n.

The all-boron/epoxy composite tubes which were fabricated with a 90-degree ply orientation failed by matrix shear between adjacent filaments. The failure torques are shown in table 5.

#### 8.4 Buckle Angle and Buckle Length

Measurements were made on the tested specimens of the angle between the axis of the tube and the line of maximum buckle depth. Measurements were also made of the length of tube which showed skin distortions due to the buckle pattern. The results of these measurements are shown in table 7. Also shown are the buckle counts predicted by the Chao analysis and from experiment.

### 9. DISCUSSION

#### 9.1 Elastic Torsion Behavior

The results shown in table 5 indicate good agreement between the predicted and experimental values of  $G_{eff}$  (eq 6) and the extension-shear coupling factors (eq 7). This agreement tends to justify the assumptions made in the elastic analysis, i.e., the axial force on the tube and the plate curvatures are zero ( $N_x = 0$ ,  $K_{ij} = 0$ ).

The agreement between the two methods of measuring shear strain (fig. 20) shows that  $\gamma_{xy}$  was essentially uniform over the tube length.

In reference 5, Whitney and Halpin observe that the radial displacement,  $w = \epsilon_z^0 R$ , does not satisfy the boundary conditions of the clamped specimen. They claim that this is of little consequence in results obtained at short distances from the ends. This assertion appears justified by the agreement between the shear strains measured over the entire gage length and those measured at the center of the specimen. This assertion appears to be valid for both long and short specimens.

#### 9.2 Computed T- $\gamma$ Curves

The torque-shear strain behavior of the short reinforced-tube specimens, Series ABS, as can be seen from figure 20e, suggests that in all of these specimens the shear yield strength of the aluminum alloy had been exceeded before buckling occurred. An attempt was made to calculate the torque-shear strain relationship using data available from the all-aluminum-alloy and the all-boron/epoxy specimens. For this calculation it was assumed that the shear strain in the aluminum alloy is equal to the shear strain in the boron/epoxy. The fraction of the total torque carried by each of the components is then a function of the cross-sectional area and the shear stress-strain relationship of the material. These calculated curves are given in figure 22. In the case of tubes reinforced with 0-degree and 90-degree

boron/epoxy plies, (figs. 22a, 22c), the differences between the calculated and experimental yield stresses are believed to be related to residual fabrication stresses in the specimen and overaging in the aluminum alloy due to the composite cure cycle. The residual stresses are caused by the differences in thermal expansion between the aluminum alloy and the boron/epoxy. In the case of the 45-degree reinforced tube, figure 22b, the large difference between the elastic slopes is a result of the coupling between shear and extension that exists in tubes reinforced with this orientation. In figure 22c, the boron stress-strain data was extrapolated to produce the dashed portion of the curve shown. As can be seen from table 5, the extension-shear coupling factor for these specimens is large when compared to factors for the other configurations tested. As the specimen is twisted, the aluminum-alloy tube does not allow the boron/epoxy laminas to extend or contract freely. As a result, the shear strain is constrained and the boron/epoxy laminas are stiffer than they would be if they acted alone. Also, the specimen is stiffer than the tubes where the coupling factor is approximately zero. Similar behavior would be expected for all ply orientations other than 0 and 90 degrees.

### 9.3 Buckling Behavior

A comparison of results for predicted and experimental buckling strengths from table 6 indicates fair agreement. The two analyses appear equally capable or incapable of predicting buckling behavior for these particular layups. However, Hayashi is not suitable for more complex layups and does not account for effects of stacking sequence, direction of loading, etc. Except for the ABS series tubes which yielded and the tubes where the material sheared before buckling occurred, both analyses have approximately the same degree of accuracy as that generally attributed to Donnell's treatment of isotropic tubes. It should be noted that presently available theories do not account for the effects of variation in surface imperfections on torsional buckling strength, as do theories of compression buckling of imperfect shells (ref. 9). There is some evidence, however, (ref. 10) that the long tubes tested in this program were insensitive to surface imperfections.

Further examination of table 6 indicates some other differences in the three behaviors (two predictions and one experimental) for the reinforced tubes. For the long tubes (ABL series), the Chao program predicts that the 90-degree orientation is strongest, followed by the 45-degree and the 0-degree orientations, in that order. The Hayashi theory predicts that the 90-degree orientation is strongest, followed by the 0-degree and the 45-degree orientations. Experimentally, the 45-degree orientation is the strongest, followed by the 90-degree and the 0-degree orientations. Similar differences can be seen in the short tubes (ABS series) where the order from strongest to weakest for the Chao theory is 45, 0, 90 degrees; for the Hayashi theory it is 90, 45, 0 degrees; and from the experiments 45, 0 90 degrees. The

experimental differences between these three orientations, however, are small relative to the increase in strength resulting from adding reinforcement. For the ABL series, for example, the buckling strength increases more than three-fold as a result of adding reinforcement while the differences between the three ply orientations is only about 17 percent.

The principal source of error in the two existing theories of torsion buckling used in this program is believed to be in the assumptions made concerning buckle pattern. If, the buckle angle is defined as the angle between the X axis of the tube and the line on the surface of minimum radial distance from the tube axis, then both of these analyses assume small buckle angles. This assumption is explicit in the Hayashi equations and is implicit in the Chao analysis where the prediction of skin buckling is governed by Donnell's approximations. From table 7 it can be seen that this assumption is generally not valid. In the case of isotropic tubes, the buckling strength is apparently not sensitive to this parameter. In the case of strongly orthotropic tubes, however, the buckling strength should be very sensitive to buckle angle. Table 7 also shows the buckle length relative to the tube length. These specimens have lengths in the region where the buckling behavior, as shown by the Hayashi analysis, is influenced by the end conditions. As a result, it was expected that the buckling behavior in the two lengths of specimens tested would be similar, i.e., that the buckles would extend the full length of the specimen. This was true only for the short tube length.

The two methods of analysis predicted only the elastic buckling strengths of the configurations tested. However the all-composite tubes, except for specimens BL-1, BL-3 and GL-1, failed by shear of the epoxy matrix. The differences in behavior of specimens BS-2 and BS-4 are noteworthy. Specimen BS-2 was tape wound, as were most of the 90-degree ply tubes. BS-4, however, was fabricated by rolling one continuous strip of broadgoods material onto a mandrel. This broadgoods material was supplied by the manufacturer with a 0.001-in (0.003-cm) fiberglass scrim cloth. The narrow tape did not have the scrim cloth. The apparent effect of the scrim cloth was to increase the material shear strength of the laminate by more than 40 percent.

#### 9.4 Post-Buckling Behavior

As shown in figure 20, only a few tubes were tested beyond initial buckling. In general it was not possible to determine the average strains in the specimen after buckling because the high localized strains which occurred at buckling destroyed many of the rosette gages and also caused the range of the troptometers to be exceeded. In such cases no further strain readings were obtained. Unreinforced specimen AL-3 and reinforced specimen ABL-3 were tested into the post buckling region and their behaviors are shown in figure 20a and 20d. As a reinforced tube buckled, the stiff boron/epoxy laminate was unable to conform to the buckled shape of the aluminum

tube. This resulted in large areas of delamination between the laminate and the aluminum. At the sharp edges of a buckle this also caused many broken boron filaments. As a result, the load-carrying capacity of the reinforced tube was reduced by approximately 50 percent. Additional loading of the specimen would have served little purpose except to increase the delaminated areas of the tube. It may be possible in future testing of reinforced tubes to prevent much of this premature delamination by introducing a layer of epoxy adhesive between the metal tube and the composite laminate.

## 10. CONCLUSIONS

Based on the work covered by this report, the following conclusions are drawn:

- 1) Laminate analysis adequately predicts the elastic behavior of composite-reinforced aluminum-alloy torsion tubes.
- 2) Existing buckling theories for anisotropic tubes have approximately the same degree of reliability as Donnell's equations for isotropic tubes for predictions of elastic buckling torques.
- 3) The 0.001-in (0.003-cm) fiberglass scrim cloth supplied by the manufacturer with the boron/epoxy broadgoods material contributes significantly to the material shear strength of the cured laminate.
- 4) The post-buckling properties of boron/epoxy reinforced tubes are seriously degraded by delaminations of the reinforcement at buckling. The use of an additional layer of adhesive between the aluminum tube and boron/epoxy composite may improve the post buckling behavior.
- 5) Buckling strength increases as a result of adding reinforcement, but the differences in strength between the three reinforcement orientations tested (0, 45 and 90 degrees) are small relative to the overall effect of adding reinforcement. Other more optimum tube layups may show significant improvements in buckling strength.

Mr. R. E. Snyder and Mr. G. F. Sushinsky gave valuable assistance in the fabrication and testing of these specimens.

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Table 1 - Dimensions of Aluminum Alloy and All Composite Material Torsion Specimens

Material type and specimen No.	Gage length		Mid surface diameter		Average thickness		Filament orientation (a)
	in	cm	in	cm	in	cm	deg
Aluminum							
AL-1	20.0	50.8	5.928	15.057	0.022	0.056	-
AL-2	20.0	50.8	5.918	15.032	.032	.081	-
AL-3	20.0	50.8	5.928	15.057	.021	.053	-
AS-1	2.00	5.08	5.928	15.057	.022	.056	-
AS-2	2.00	5.08	5.928	15.057	.022	.056	-
AS-3	2.00	5.08	5.905	14.999	.045	.114	-
Glass/epoxy							
GL-1	20.0	50.8	6.040	15.342	0.021	0.053	0
GL-2	20.0	50.8	6.040	15.342	.021	.053	90
Boron/epoxy							
BL-1	20.0	50.8	6.104	15.504	0.022	0.056	0
BS-1	2.00	5.08	6.074	15.428	.023	.058	0
BS-1A	2.00	5.08	6.074	15.428	.023	.058	0
BS-1B	2.00	5.08	6.074	15.403	.023	.058	0
BL-3	20.0	50.8	6.064	15.403	.023	.058	45
BS-3	2.00	5.08	6.058	15.387	.023	.058	45
BL-2	20.0	50.8	6.060	15.392	.018	.046	90
BL-2A	15.0	38.1	6.060	15.392	.018	.046	90
BS-2	2.00	5.08	6.030	15.316	.018	.046	90
BS-4	2.0	5.08	6.008	15.260	.022	.056	90

(a) Filament orientation is measured with respect to the X axis.

Table 2 - Dimensions of Composite-Reinforced Aluminum-Alloy Torsion Specimens

Specimen No.	Gage length		Mid surface diameter		$t_c$		$t_a$		Total thickness		Orientation deg
	in	cm	in	cm	in	cm	in	cm	in	cm	
ABL-1	20.0	50.8	5.951	15.116	0.025	0.064	0.023	0.058	0.048	0.122	0
ABL-2	20.0	50.8	5.950	15.113	.022	.056	.023	.058	.045	.114	90
ABL-3	20.0	50.8	5.950	15.113	.025	.064	.021	.053	.046	.117	0
ABL-4	20.0	50.8	5.949	15.110	.020	.051	.022	.056	.042	.107	90
ABL-5	20.0	50.8	5.951	15.116	.024	.061	.022	.056	.046	.117	45
ABL-6	20.0	50.8	5.950	15.113	.023	.058	.022	.056	.045	.114	45
ABS-1	2.00	5.08	5.950	15.113	.022	.056	.023	.058	.045	.114	0
ABS-2	2.00	5.08	5.949	15.110	.020	.051	.022	.056	.042	.107	90
ABS-3	2.00	5.08	5.950	15.113	.022	.056	.022	.056	.044	.112	0
ABS-4	2.00	5.08	5.950	15.113	.023	.058	.022	.056	.045	.114	0
ABS-5	2.00	5.08	5.950	15.113	.022	.056	.023	.058	.045	.114	45
ABS-6	2.00	5.08	5.950	15.113	.023	.058	.022	.056	.045	.114	45
ABS-7	2.00	5.08	5.946	15.103	.015	.038	.022	.056	.037	.094	90
AGL-1	20.0	50.8	5.950	15.113	.023	.058	.023	.058	.046	.117	0

Table 3 - Results of Volume Fraction Measurements

Tube No.	Ply angle Degrees	Boron/epoxy thickness in cm		Volume fraction	
				Boron percent	Epoxy + scrim + void percent
ABL-1	0	0.025	0.064	32.7	67.3
ABL-3	0	0.025	0.064	36.4	63.6
ABL-2	90	0.022	0.056	52.8	47.2
ABL-4	90	0.020	0.051	56.5	43.5
ABL-5	45	0.024	0.061	45.0	55.0
ABL-6	45	0.023	0.058	45.8	54.2
ABS-3	0	0.022	0.056	43.3	56.7
ABS-4	0	0.023	0.058	40.8	59.2
ABS-2	90	0.020	0.051	61.9	38.1
ABS-7	90	0.015	0.038	58.5	41.5
ABS-5	45	0.022	0.056	43.4	56.6
ABS-6	45	0.023	0.058	42.5	57.5
BL-2	90	0.018	0.046	56.3	43.7
BS-2	90	0.018	0.046	50.6	49.4
BS-4	90	0.022	0.056	35.1	64.9

Table 4 - Average Material Properties

Material	Young's Modulus in the direction of		Principal in plane Poisson's Ratio
	Major axis	Minor axis	
	1bf/in <sup>2</sup>	1bf/in <sup>2</sup>	
Aluminum alloy	10.2 x 10 <sup>6</sup>	10.2 x 10 <sup>6</sup>	0.33
Boron/epoxy	31	2.5	.21
Glass/epoxy	5.0	1.4	.25(a)

(a) This property was not measured in this program but is reported by the manufacturer.

Table 5 - Results of Elastic Behavior Measurements

Tube No.	Ply angle degrees	Laminate analysis					Experimental			
		Coupling coefficient			Shear modulus		Coupling coef- ficient $\frac{\epsilon_x + \epsilon_y}{2\gamma}$	Shear modulus		
		$\epsilon_x^o/\gamma$	$\epsilon_y^o/\gamma$	$\frac{\epsilon_x + \epsilon_y}{2\gamma}$	$G_{eff}$	$G_{eff}$		$G_{eff}$	$G_{eff}$	
					1bf/in <sup>2</sup>	N/m <sup>2</sup>	1bf/in <sup>2</sup>	N/m <sup>2</sup>		
AL-1	-	0.000	0.000	0.000	3.9 x 10 <sup>6</sup>	27 x 10 <sup>9</sup>	0.016	3.7 x 10 <sup>6</sup>	26 x 10 <sup>9</sup>	
AL-2	-	0.000	0.000	0.000	3.9	27	-0.005	3.9	27	
AL-3	-	0.000	0.000	0.000	3.9	27	-0.002	4.0	28	
AS-1	-	0.000	0.000	0.000	3.9	27	0.003	3.5	24	
AS-2	-	0.000	0.000	0.000	3.9	27	0.000	3.8	26	
AS-3	-	0.000	0.000	0.000	3.9	27	-0.008	4.0	28	
ABL-1	0	0.000	0.000	0.000	2.1	15	0.013	2.0	14	
ABL-3	0	0.000	0.000	0.000	2.1	14	0.003	2.1	14	
ABL-2	90 <sup>(b)</sup>	0.002	-0.005	-0.002	2.2	15	-0.003	1.9	13	
ABL-4	90 <sup>(b)</sup>	0.002	-0.006	-0.002	2.3	16	-0.004	2.4	16	
ABL-5	45	-0.209	-0.209	-0.209	4.1	28	-0.211	4.2	29	
ABL-6	45	-0.209	-0.209	-0.209	4.2	29	-0.214	4.2	29	
ABS-1	0	0.000	0.000	0.000	2.3	16	-0.002	1.9	13	
ABS-3	0	0.000	0.000	0.000	2.2	15	0.002	2.3	16	
ABS-4	0	0.000	0.000	0.000	2.2	15	-0.012	2.1	15	
ABS-2	90 <sup>(b)</sup>	0.002	-0.006	-0.002	2.3	16	0.002	2.4	17	
ABS-7	90 <sup>(b)</sup>	0.002	-0.006	-0.002	2.6	18	-0.034	2.8	19	
ABS-5	45	-0.204	-0.205	-0.204	4.3	30	-0.220	4.1	28	
ABS-6	45	-0.209	-0.209	-0.209	4.2	29	-0.224	4.5	31	
BL-1	0	0.000	0.000	0.000	0.59	4.1	-0.021	0.60	4.1	
BL-2	90 <sup>(b)</sup>	0.004	-0.008	-0.002	0.65	4.4	-0.059	0.80	5.5	
BL-3	45	-0.410	-0.410	-0.410	2.0	14	-0.380	2.0	14	
BS-1	0	0.000	0.000	0.000	0.58	4.0	-0.008	0.56	3.9	
BS-1A	0	0.000	0.000	0.000	0.58	4.0	0.000	0.66	4.5	
BS-1B	0	0.000	0.000	0.000	0.58	4.0	0.027	0.62	4.3	
BS-2	90 <sup>(b)</sup>	0.004	0.008	0.002	0.64	4.4	-0.023	0.66	4.5	
BS-4	90	0.000	0.000	0.000	0.58	4.0	0.009	0.54	3.7	
BS-3	45	-0.410	-0.410	-0.410	2.0	14	-0.400	2.1	14	
GL-1	0	0.000	0.000	0.000	0.44	3.0	0.013	0.56	3.9	
GL-2	90	0.000	0.000	0.000	0.44	3.0	-0.028	0.50	3.5	
AGL-1	0	0.000	0.000	0.000	2.1	15	0.001	2.3	16	
BL-2A <sup>(a)</sup>	90 <sup>(b)</sup>	0.000	0.000	0.000	0.65	4.1	0.015	0.84	5.8	

(a) See specimen BL-2. Specimen BL-2A has a length of 15 in.

(b) Nominal 90 degree ply orientation. These specimens are tape wound. Actual ply orientation angle, 89.5 degrees. These specimens do not have a 0.001-in (0.003-cm) scrim cloth per ply.

Table 6 - Buckling Strength Results

Tube No.	Ply angle degrees	Aluminum-alloy thickness		Composite thickness	Predicted buckling torque			Experimental buckling torque			Knockdown factors (a)					
		cm			Chao		Maximum		At strain gage reversal		Strain reversal		Maximum			
		in	cm		lbf-in	N-m	lbf-in	N-m	lbf-in	N-m	Chao	Hayashi	Chao	Hayashi		
AL-1	-	0.022	0.056	-	8,400	950	8,600	970	7,100	810	6,800	770	0.81	0.79	0.84	0.82
AL-3	-	.021	.053	-	7,600	860	7,800	880	7,400	840	7,000	790	.92	.90	.97	.95
AL-2	-	.032	.081	-	19,200	2,170	20,000	2,260	19,000	2,150	19,000	2,150	.99	.95	.99	.95
AS-1	-	.022	.056	-	28,900	3,270	35,100	3,970	20,500	2,320	20,000	2,260	.69	.57	.71	.58
AS-2	-	.022	.056	-	28,900	3,270	35,100	3,970	23,000	2,600	22,000	2,490	.76	.63	.80	.66
AS-3	-	.045	.114	-	167,000	18,800	206,000	23,200	>66,000	>7,460	(c)	(c)	-	-	-	-
BL-1	0	-	-	0.022	2,980	337	4,800	540	2,800	310	2,800	310	.94	.58	.94	.58
BL-2	90(b)	-	-	.018	3,630	410	7,100	800	>1,800	>200	(f)	(f)	-	-	-	-
BL-3	45	-	-	.023	5,250	593	4,300	490	3,400	390	3,300	370	.63	.77	.65	.79
BL-2A	90(b)	-	-	.018	5,300	600	6,300	720	>3,500	>400	(f)	(f)	-	-	-	-
BS-1	0	-	-	.023	13,900	1,570	17,400	1,960	(e)	(e)	(e)	(e)	-	-	-	-
BS-1A	0	-	-	.023	13,900	1,570	17,400	1,960	(e)	(e)	(e)	(e)	-	-	-	-
BS-1B	0(b)	-	-	.023	13,900	1,570	17,400	1,960	7,800	880	7,800	880	.56	.45	.56	.45
BS-2	90	-	-	.018	39,700	806	39,700	4,490	>3,500	>400	(f)	(f)	-	-	-	-
BS-4	90	-	-	.022	13,600	1,540	51,700	5,840	>6,000	>680	(f)	(f)	-	-	-	-
BS-3	45	-	-	.023	33,500	3,790	20,200	2,280	9,000	1,020	9,000	1,020	.27	.45	.27	.45
ABL-1	0	.023	.058	.025	30,000	3,390	31,800	3,590	24,200	2,790	23,500	2,660	.78	.74	.81	.76
ABL-3	0	.021	.053	.025	27,100	3,060	28,900	3,260	23,800	2,680	23,800	2,660	.88	.82	.88	.82
ABL-2	90(b)	.023	.058	.022	34,700	3,920	37,100	4,200	26,200	2,960	24,500	2,770	.71	.66	.76	.71
ABL-4	90(b)	.022	.056	.020	30,000	3,390	36,200	4,100	28,600	3,230	28,000	3,160	.93	.77	.95	.79
ABL-5	45	.022	.056	.024	32,500	3,670	25,500	2,880	28,600	3,230	28,000	3,160	.86	1.10	.88	1.12
ABL-6	45	.022	.056	.023	31,300	3,540	24,800	2,800	27,000	3,050	27,000	3,050	.86	1.09	.86	1.09
ABS-1	0	.023	.058	.022	127,000	14,400	120,000	13,500	40,700	4,600	(d)	(d)	-	-	-	-
ABS-3	0	.022	.056	.022	121,000	13,700	113,000	12,800	42,000	4,740	42,000	4,740	.35	.37	.35	.37
ABS-4	0	.022	.056	.023	117,000	13,900	117,000	13,200	43,000	4,860	43,000	4,860	.35	.37	.35	.37
ABS-2	90(b)	.022	.056	.020	97,100	11,000	204,000	23,000	36,200	4,100	36,200	4,100	.37	.18	.37	.18
ABS-7	90(b)	.022	.056	.015	115,000	13,000	212,000	24,000	37,600	4,250	37,500	4,240	.33	.18	.33	.18
ABS-5	45	.023	.058	.022	163,000	18,400	126,000	14,200	56,000	6,330	54,500	6,160	.33	.43	.34	.44
ABS-6	45	.022	.056	.023	158,000	17,900	123,000	13,900	62,000	7,000	59,500	6,720	.38	.48	.39	.50
GT-1	0	-	-	.021	1,320	149	1,700	190	1,200	130	1,100	120	.83	.65	.91	.70
GT-2	90	-	-	.021	2,070	233	2,400	270	1,900	210	1,700	190	.81	.70	.92	.79
AGL-1	0	.023	.058	.023	17,100	1,930	18,100	2,040	16,500	1,860	16,500	1,860	.96	.91	.96	.91

(a) Knockdown factor is defined as the ratio of experimental torque at strain reversal or maximum torque to predicted buckling torque.

(b) Nominal 90 degrees, measured 89.5 degrees.

(c) Machine capacity exceeded.

(d) Buckled after one minute at maximum load.

(e) End cap debonded before failure.

(f) Material failure.

Table 7 - Results of Buckle Angle and Buckle Length Measurements

Tube No.	Ply angle degrees	Gage length		Buckle count		Buckle angle degrees	Buckle length	
		in	cm	Predicted <sup>(b)</sup>	Measured		in	cm
AL-1	-	20	51	5	(c)	24	8	20
AL-2	-	20	51	5	5 <sup>(a)</sup>	24	13.5	34
AL-3	-	20	51	5	5	21	10.5	27
AS-1	-	2	5	12	12	52	2	5
AS-2	-	2	5	12	12	53	2	5
AS-3	-	2	5	9	(c)	(d)	(d)	(d)
ABL-1	0	20	51	5	5 <sup>(a)</sup>	15	12.5	32
ABL-3	0	20	51	5	5	13	12.5	25
ABL-2	90	20	51	4	(c)	35	10	32
ABL-4	90	20	51	4	(c)	35	11	28
ABL-5	45	20	51	4	5 <sup>(a)</sup>	45	8	20
ABL-6	45	20	51	4	4	45	18	46
ABS-1	0	2	5	10	(c)	(d)	(d)	(d)
ABS-3	0	2	5	10	9	58	2	5
ABS-4	0	2	5	10	9 <sup>(a)</sup>	57	2	5
ABS-2	90	2	5	9	11	48	2	5
ABS-7	90	2	5	10	9	50	2	5
ABS-5	45	2	5	7	13	45	2	5
ABS-6	45	2	5	7	13	45	2	5
BL-1	0	20	51	7	6	20	(d)	(d)
BL-3	45	20	51	4	(c)	45	(d)	(d)
BS-1B	0	2	5	14	12 <sup>(a)</sup>	27	2	5
BS-3	45	2	5	7	23 <sup>(a)</sup>	45	2	5
GL-1	0	20	51	6	7	(d)	(d)	(d)
AGL-1	0	20	51	5	5 <sup>(a)</sup>	20	10.5	27

(a) Estimated from incomplete buckle pattern.

(b) According to Chao program.

(c) Testing discontinued when first buckle appeared.

(d) No buckle visible for measurement after test completed.

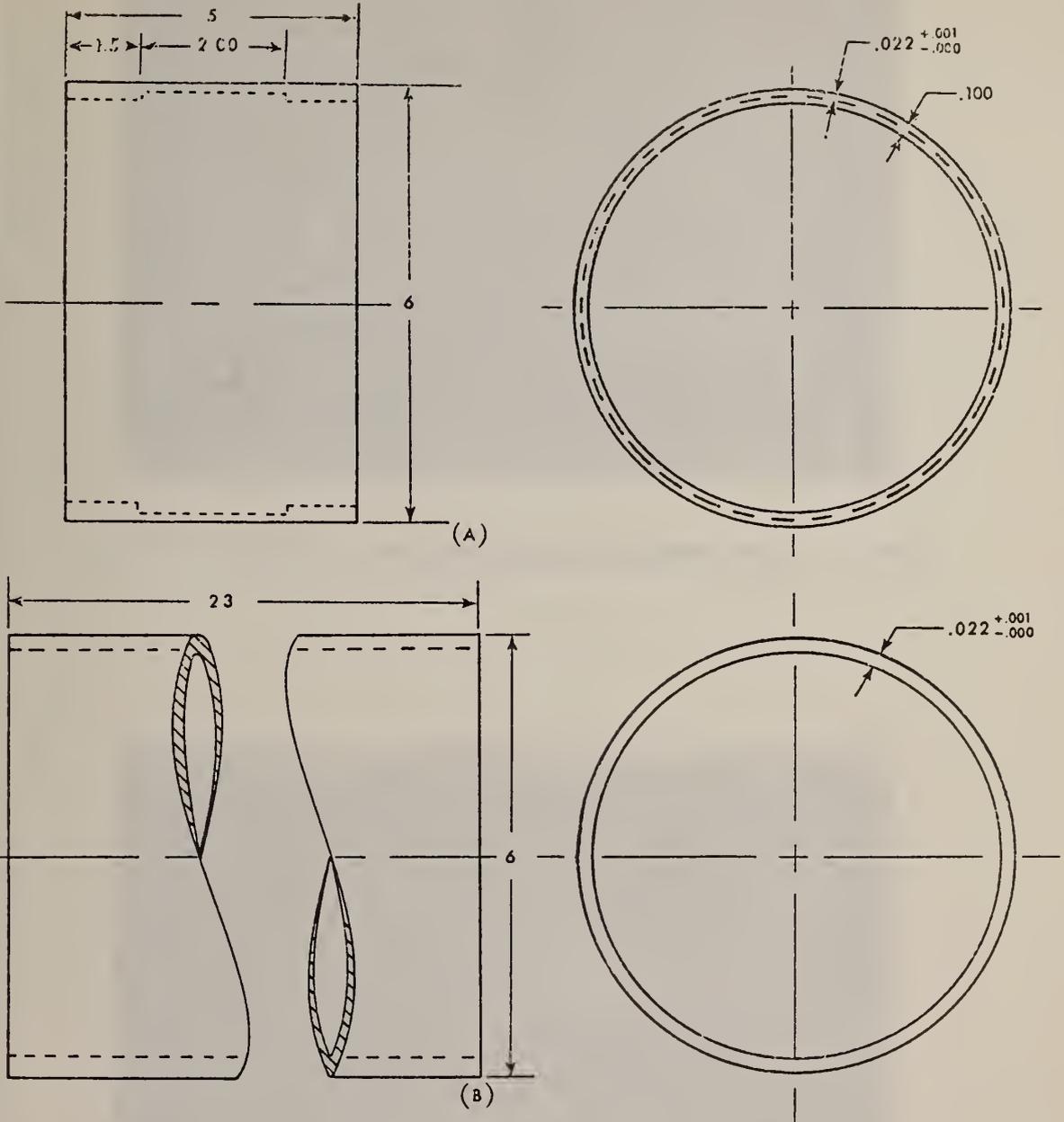


Figure 1. Aluminum-alloy tube specimens. All dimensions are given in inches (1 in = 2.54 cm).



Figure 2. Wood mandrel covered with heat shrinkable tubing.

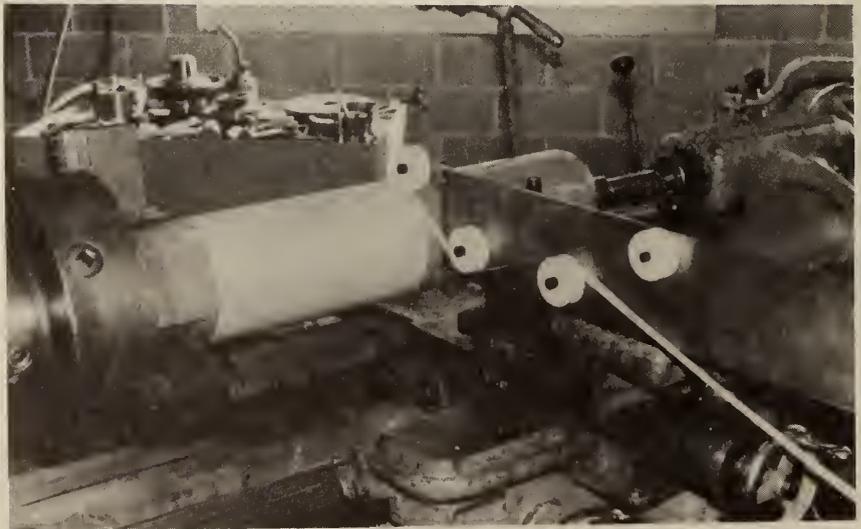


Figure 3. Tape winding glass/epoxy tube.



Figure 4. Tape winding boron/epoxy tube



Figure 5. Lay-up of 0-degree reinforced aluminum specimen.



Figure 6. Lay-up of 45-degree specimen.



Figure 7. Placement of release glass cloth over lay-up.



Figure 8. Placement of glass bleeder cloth over lay-up.



Figure 9. Installation of heat shrinkable tubing over the part prior to cure cycle.

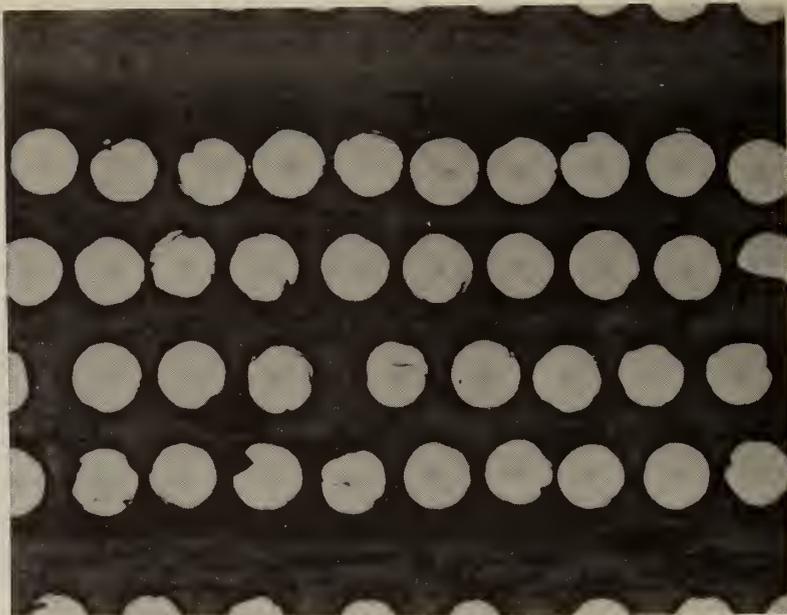


Figure 10a. Specimen ABS-5 (100x).

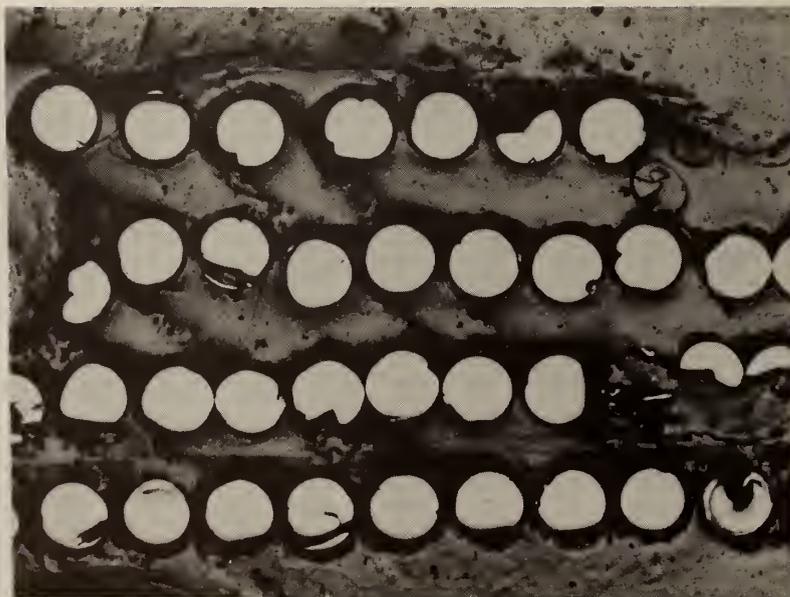


Figure 10b. Specimen ABL-3 (100x).

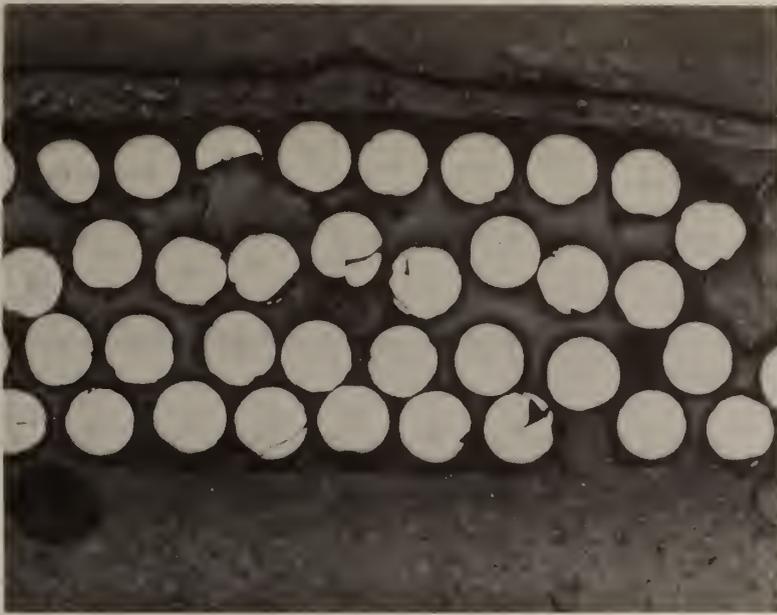
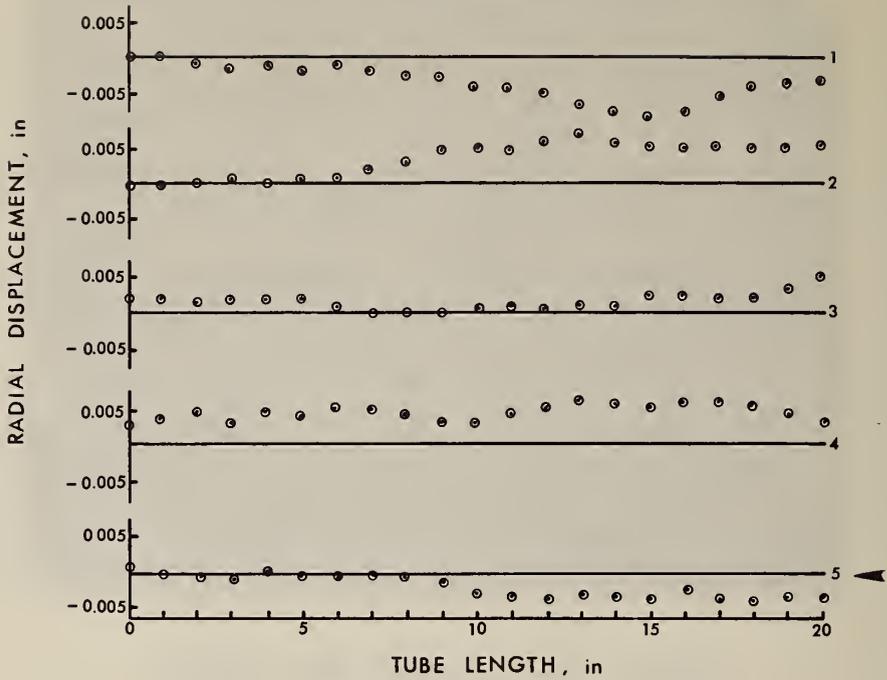


Figure 10c. Specimen ABS-7 (100x).

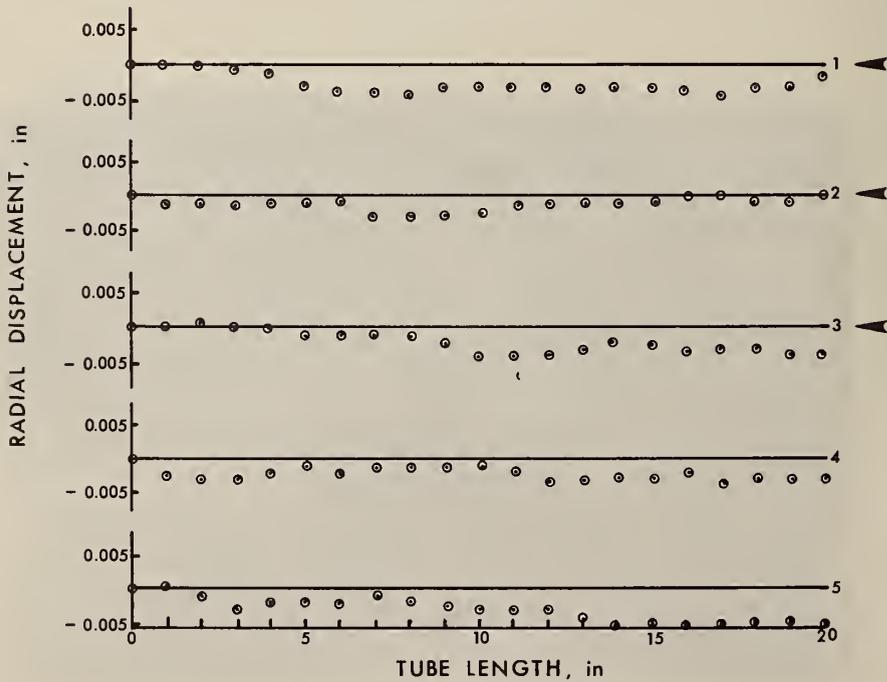


Figure 11. Set up for profile measurements.

SPECIMEN AL-3



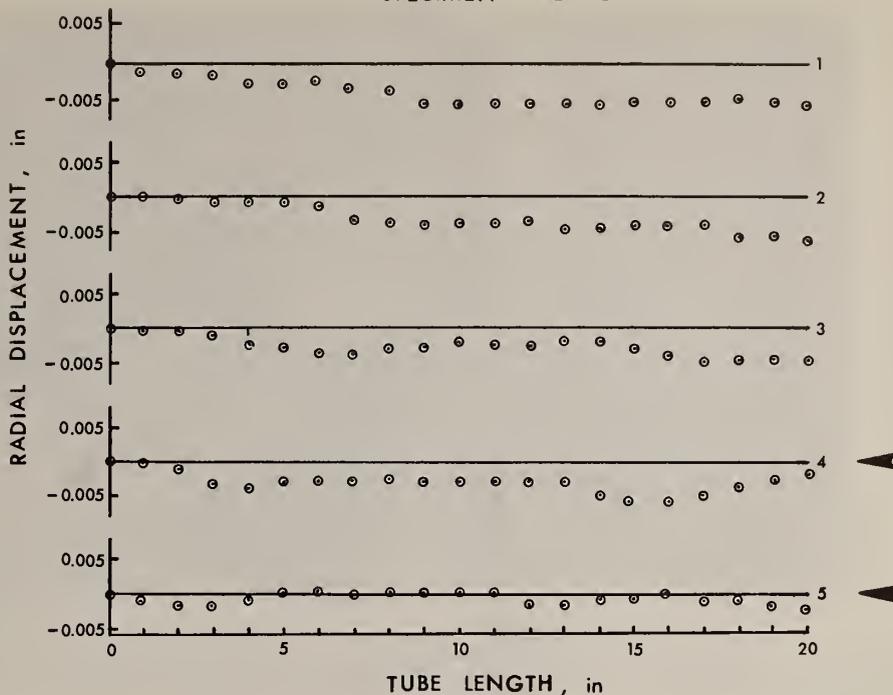
SPECIMEN ABL-5



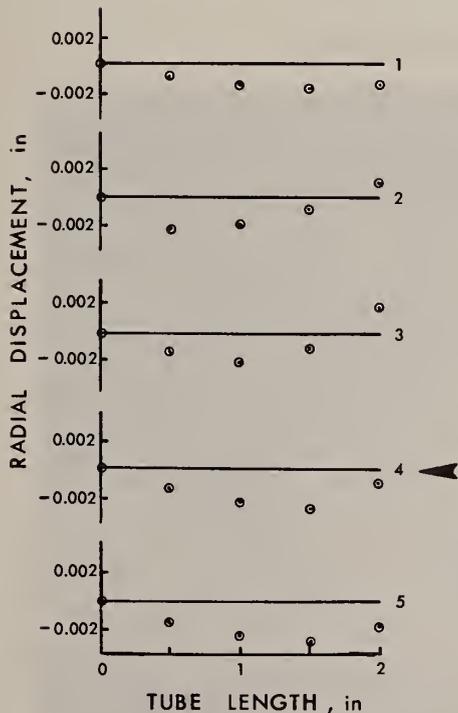
◀ First buckle as indicated by strain gage reversal

Figure 12a. Surface imperfection profile measurements.

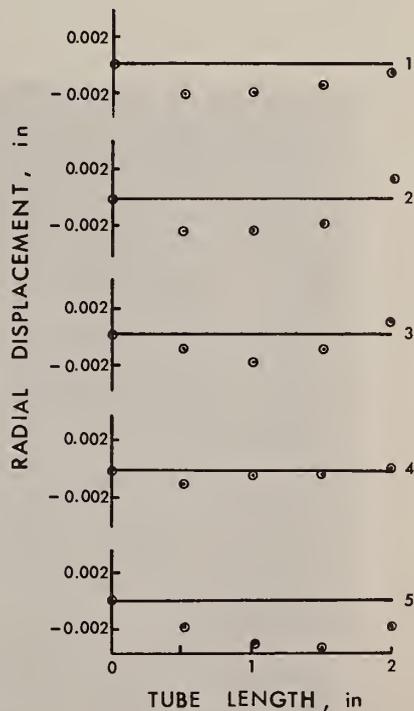
SPECIMEN ABL-6



SPECIMEN AS-2



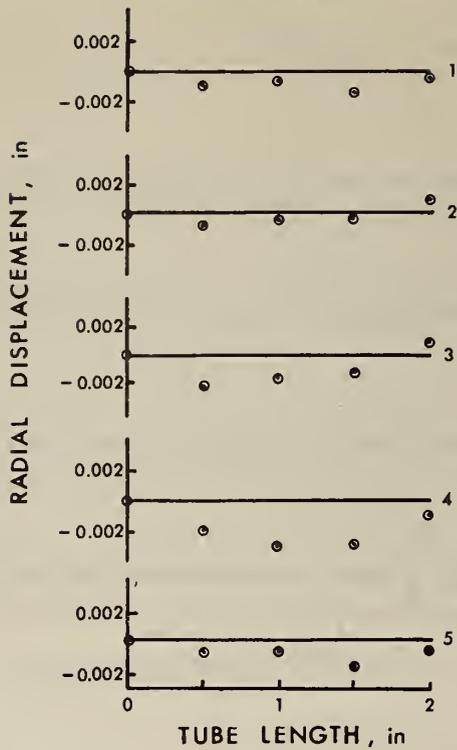
SPECIMEN ABS-3



◀ First buckle as indicated by strain gage reversal

Figure 12b. Surface imperfection profile measurements (continued).

SPECIMEN ABS-4



SPECIMEN ABS-6

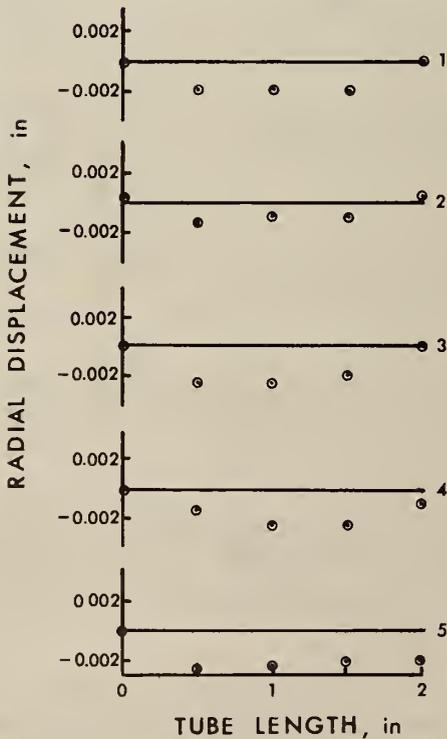


Figure 12c. Surface imperfection profile measurements (continued).



Figure 13. Typical strain gage installation on exterior surface of a tube.



Figure 14. Typical strain gage installation on interior surface of a tube.

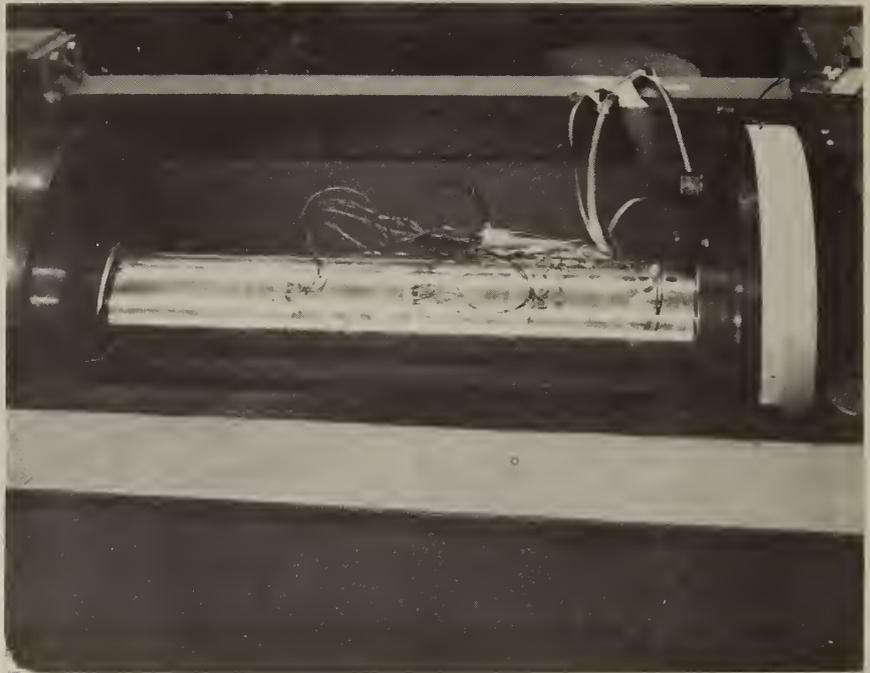
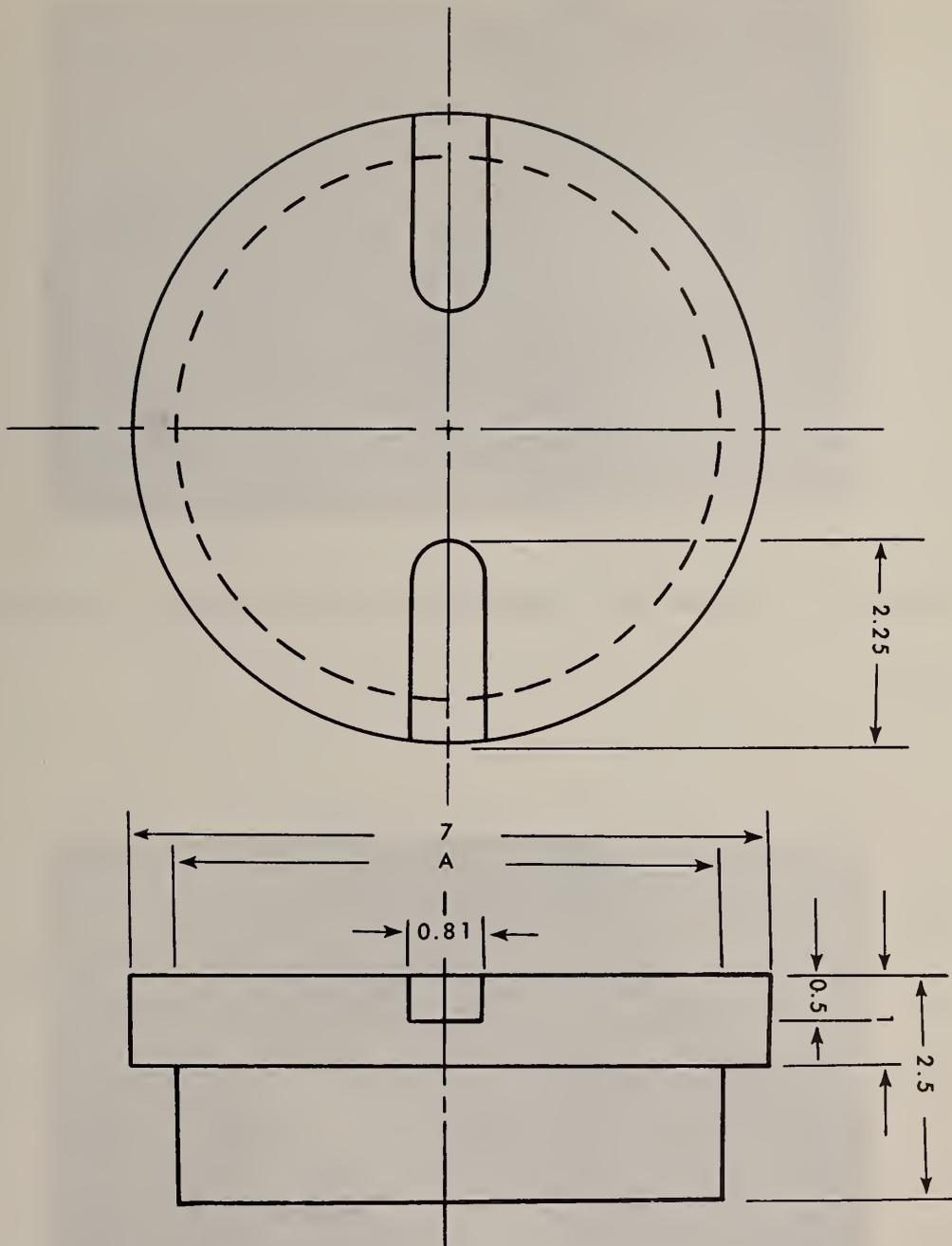


Figure 15. Typical strain gage installation on specimen AL-1.



Figure 16. Troptometer set-up showing auto-collimators and mirrors.



A. Nominal 6 in. — Machined to fit individual specimen

Figure 17. End fixtures. All dimensions are given in inches (1 in = 2.54 cm).

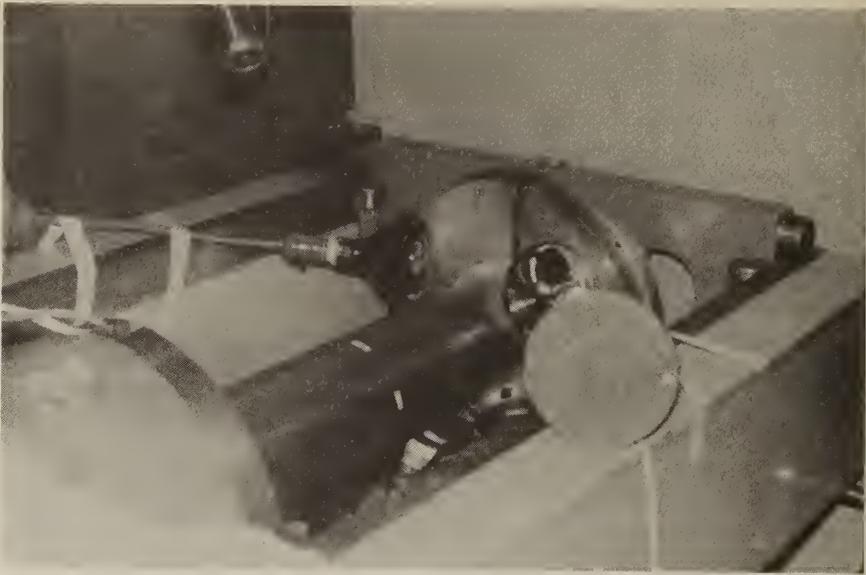


Figure 18. Radiant heat lamps used to bond end caps to specimen.

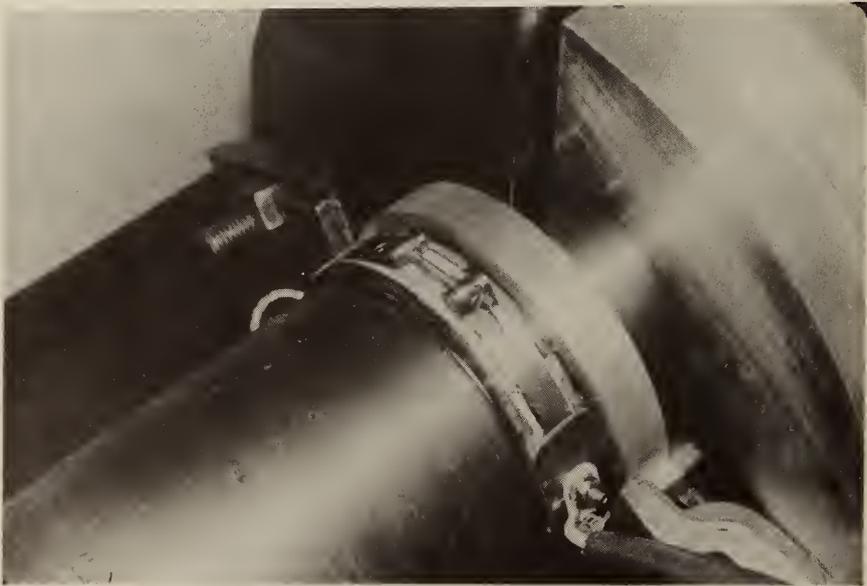


Figure 19. Heated collars used to bond end caps to specimen.

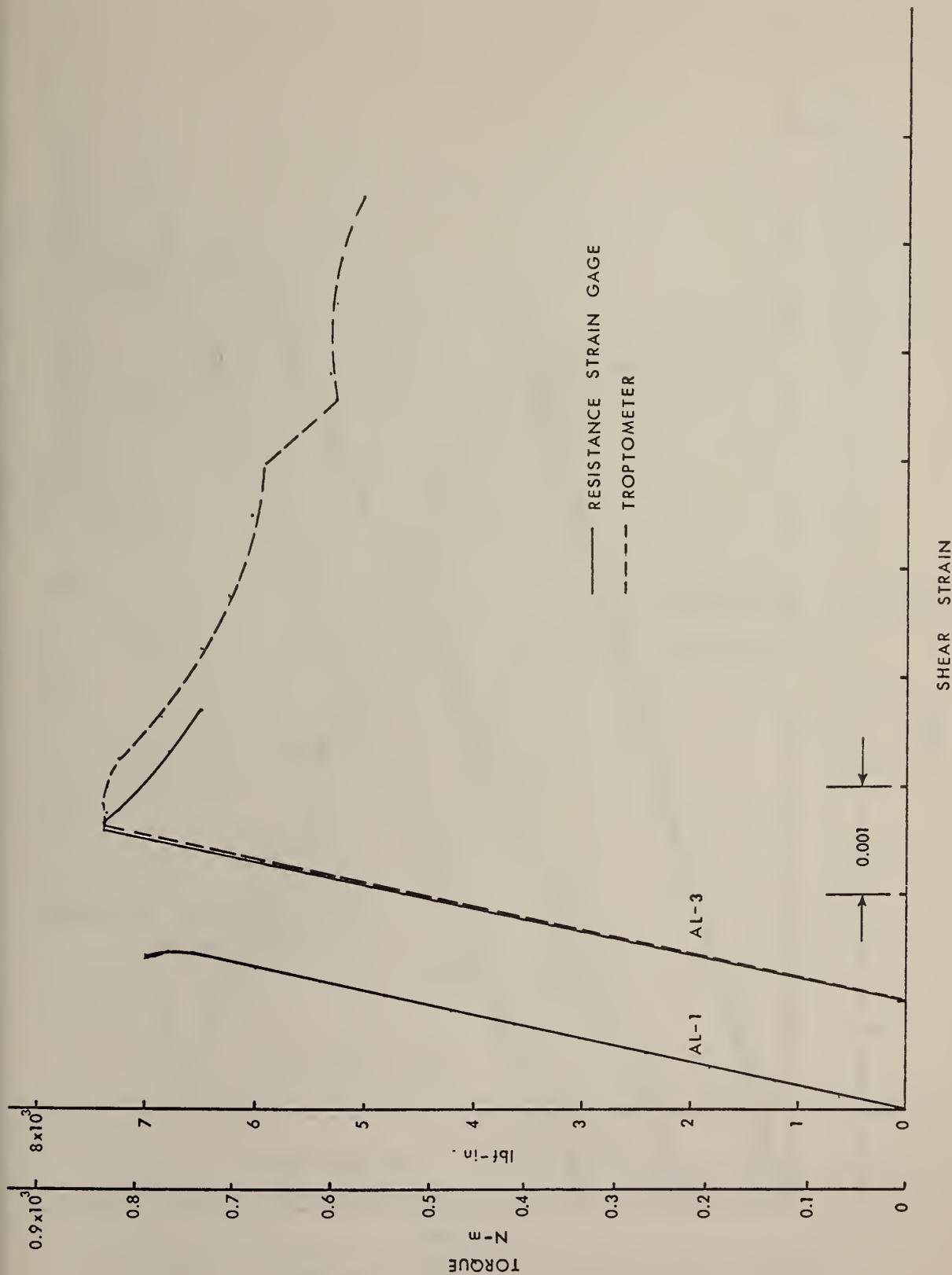


Figure 20a. Torque-shear strain curves for specimens AL-1, and AL-3.

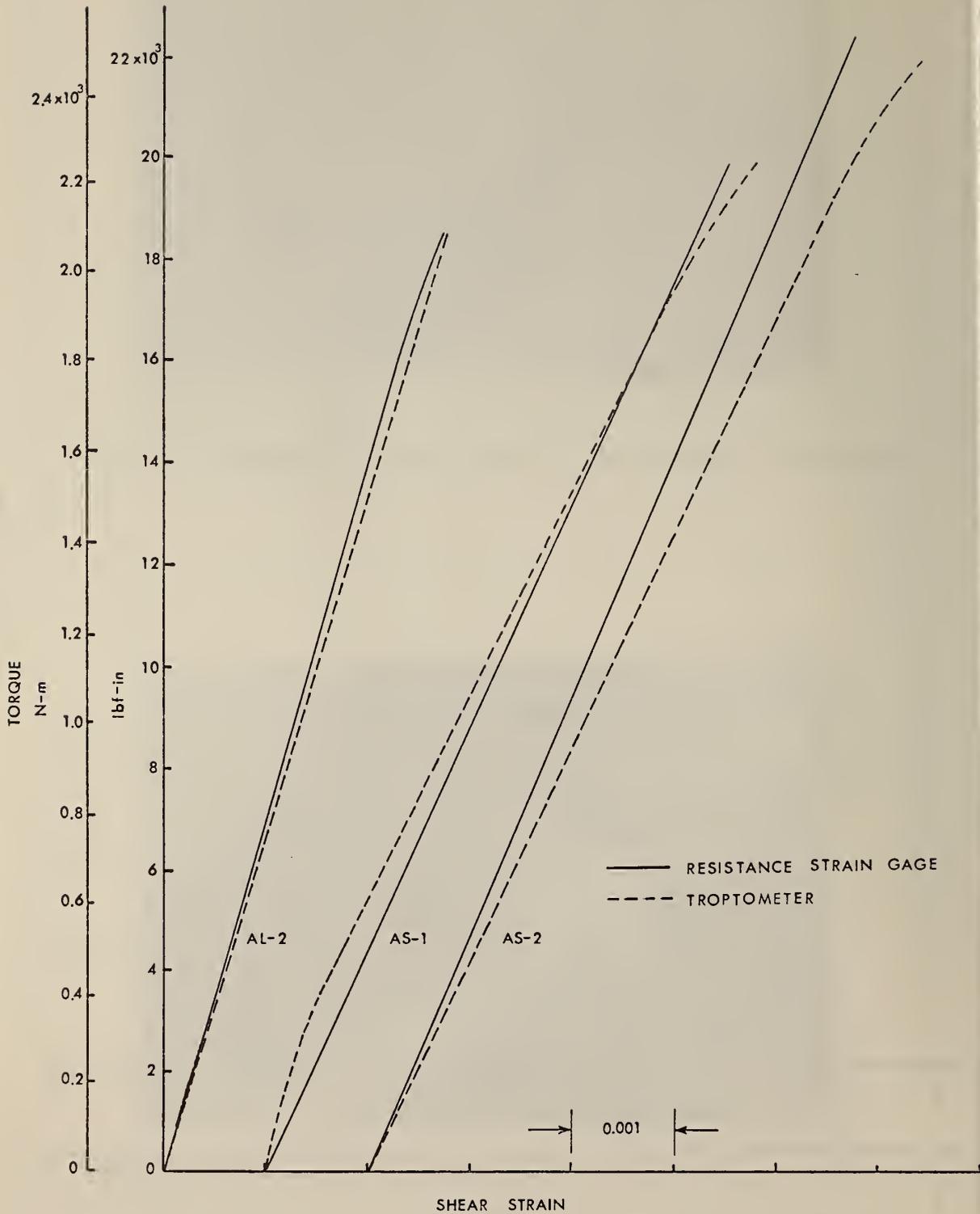


Figure 20b. Torque-shear strain curves for specimens AL-2, AS-1 and AS-2

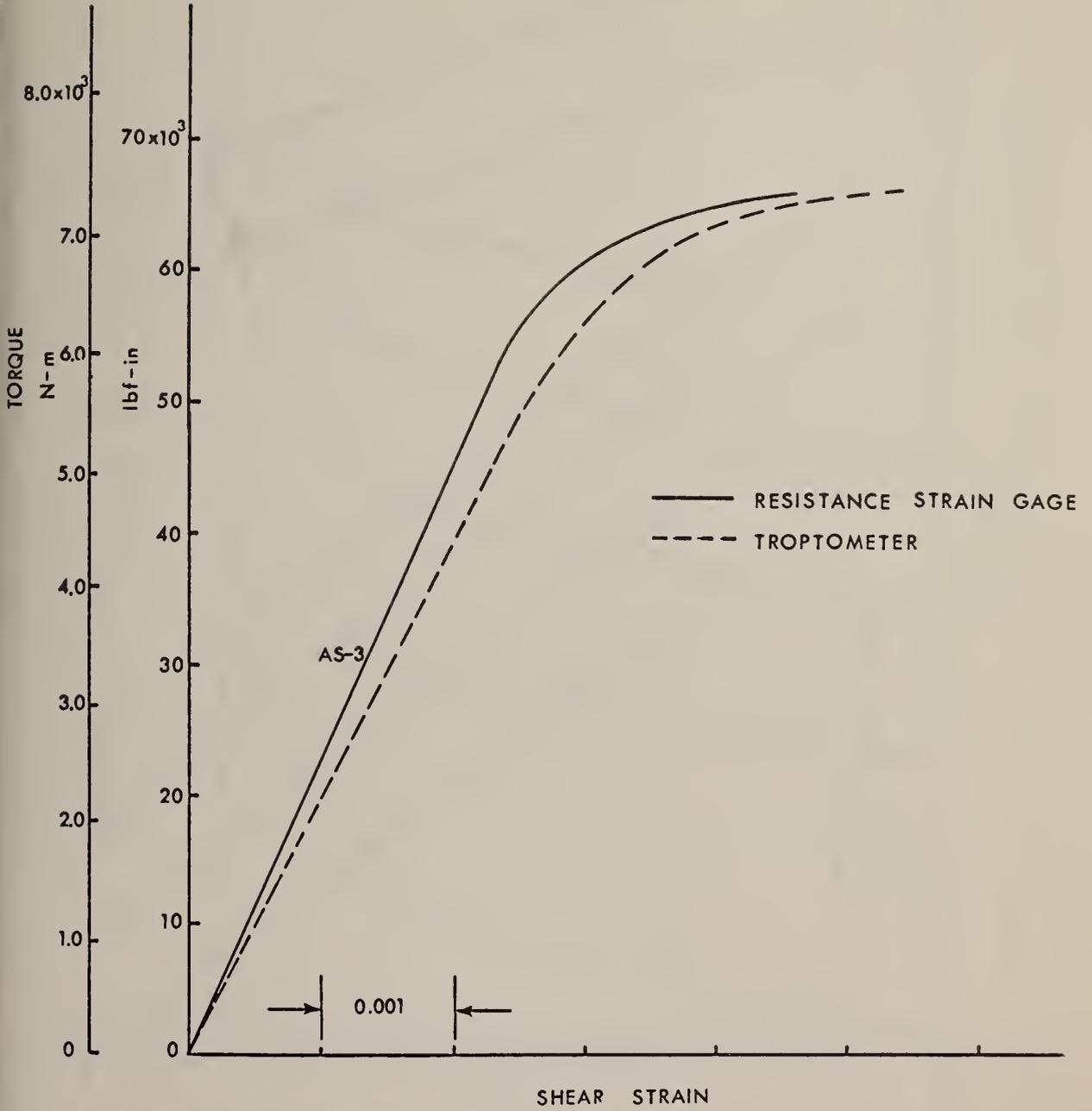


Figure 20c. Torque-shear strain curve for specimen AS-3.

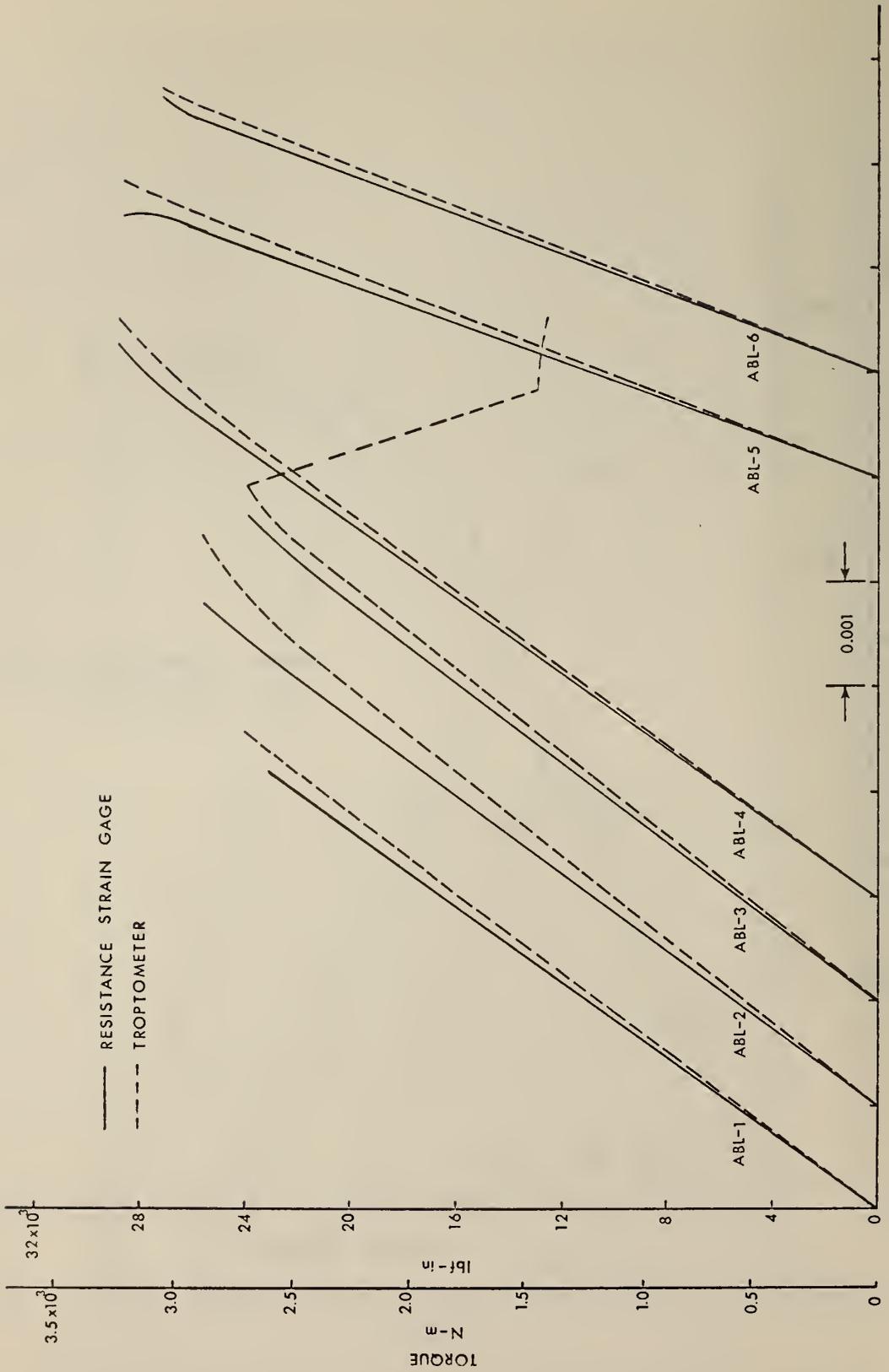


Figure 20d. Torque-shear strain curves for specimens ABL-1, ABL-2, ABL-3, ABL-4, ABL-5 and ABL-6.

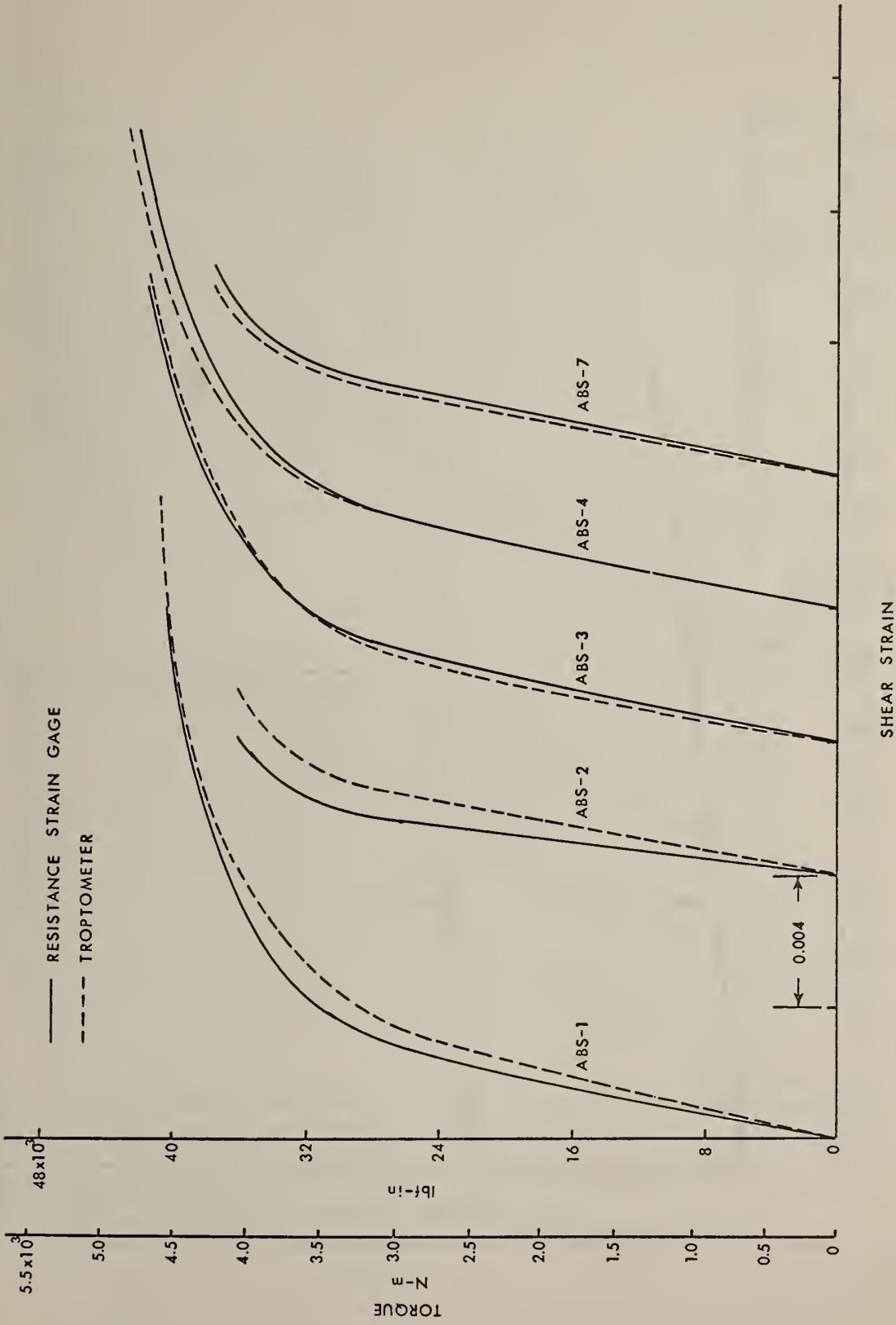


Figure 20e. Torque-shear strain curves for specimens ABS-1, ABS-2, ABS-3, ABS-4, and ABS-7.

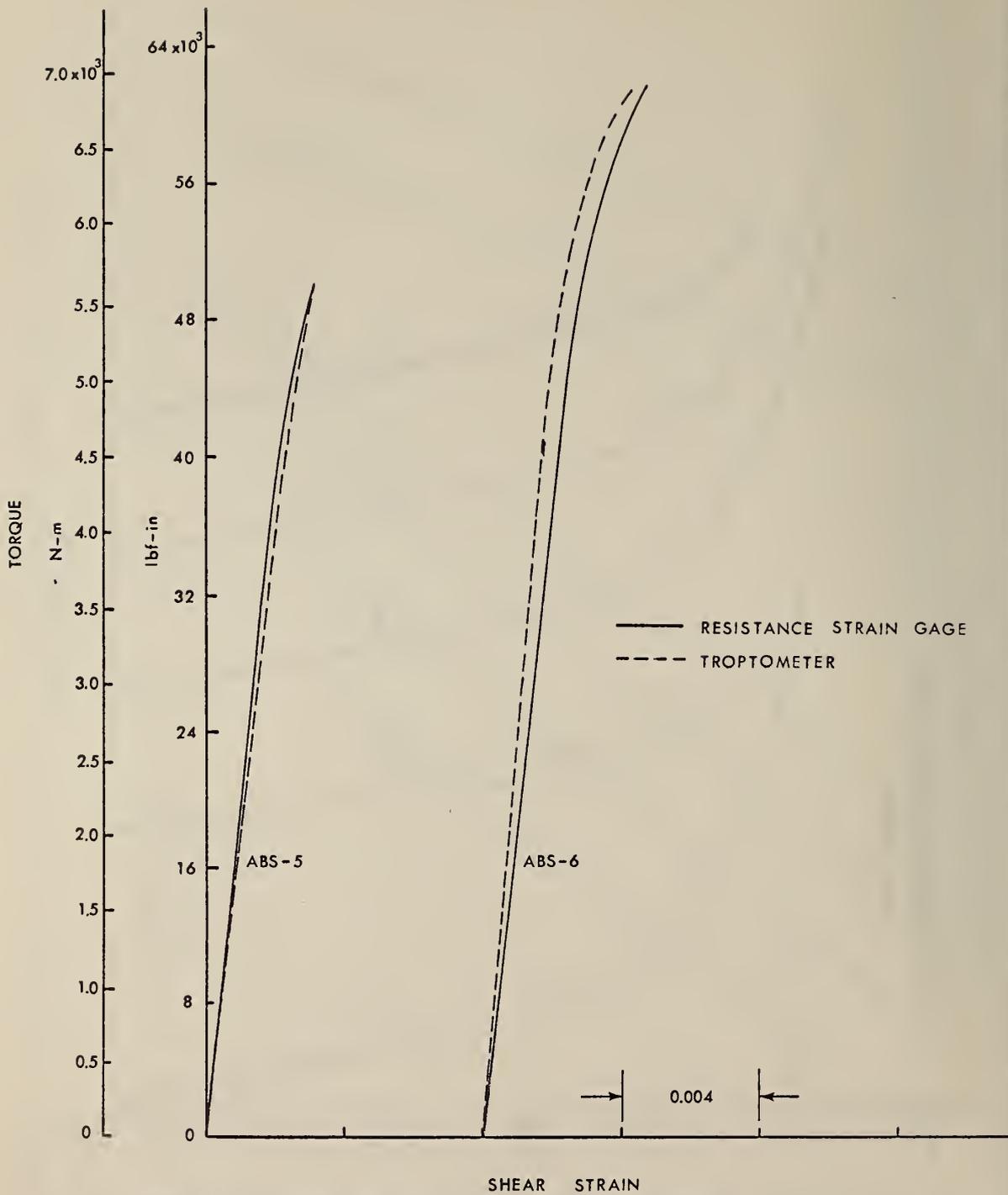
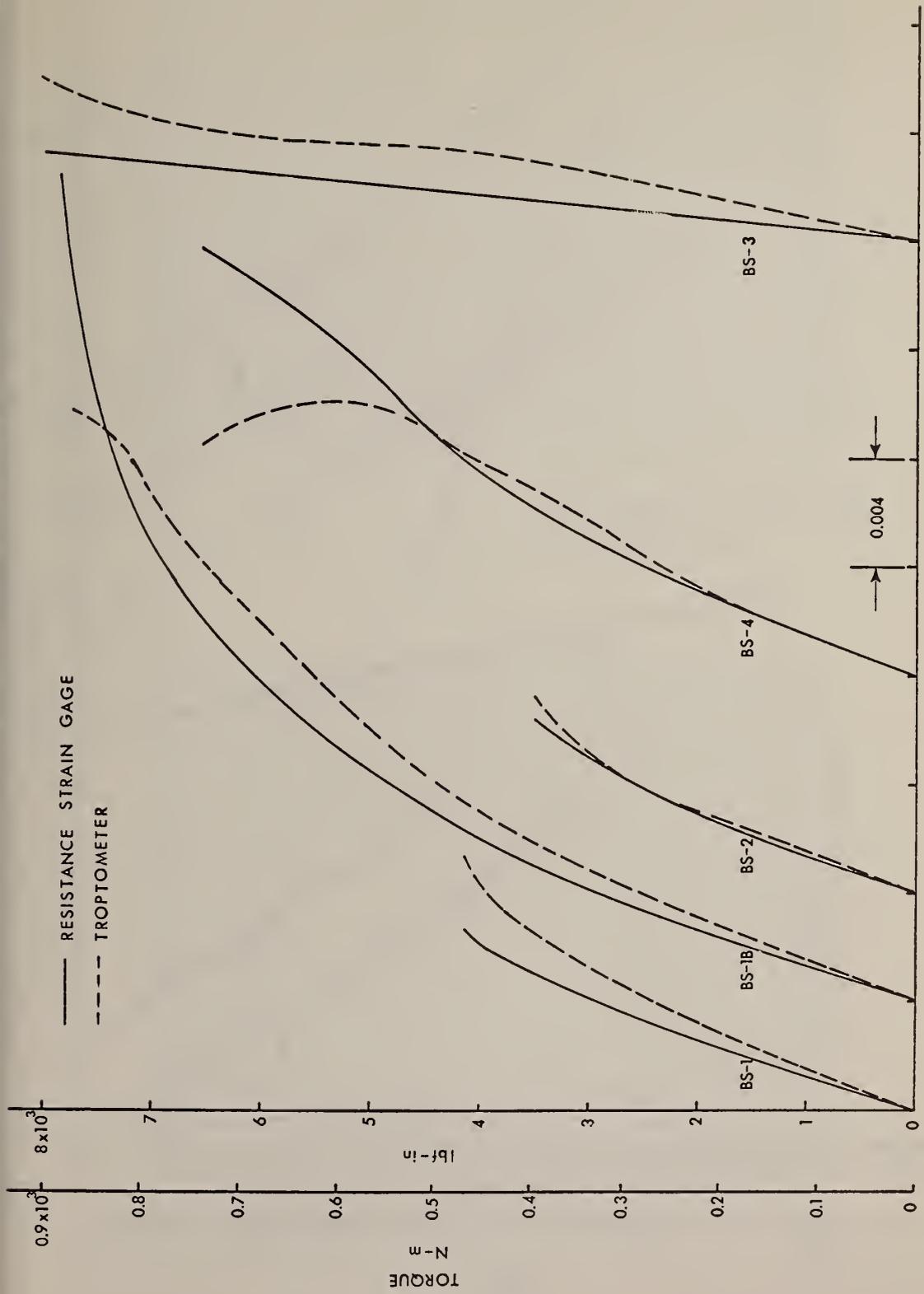


Figure 20f. Torque-shear strain curves for specimens ABS-5 and ABS-6



SHEAR STRAIN

Figure 20g. Torque-shear strain curves for specimens BS-1, BS-1B, BS-2, BS-4, BS-3 and BS-4.

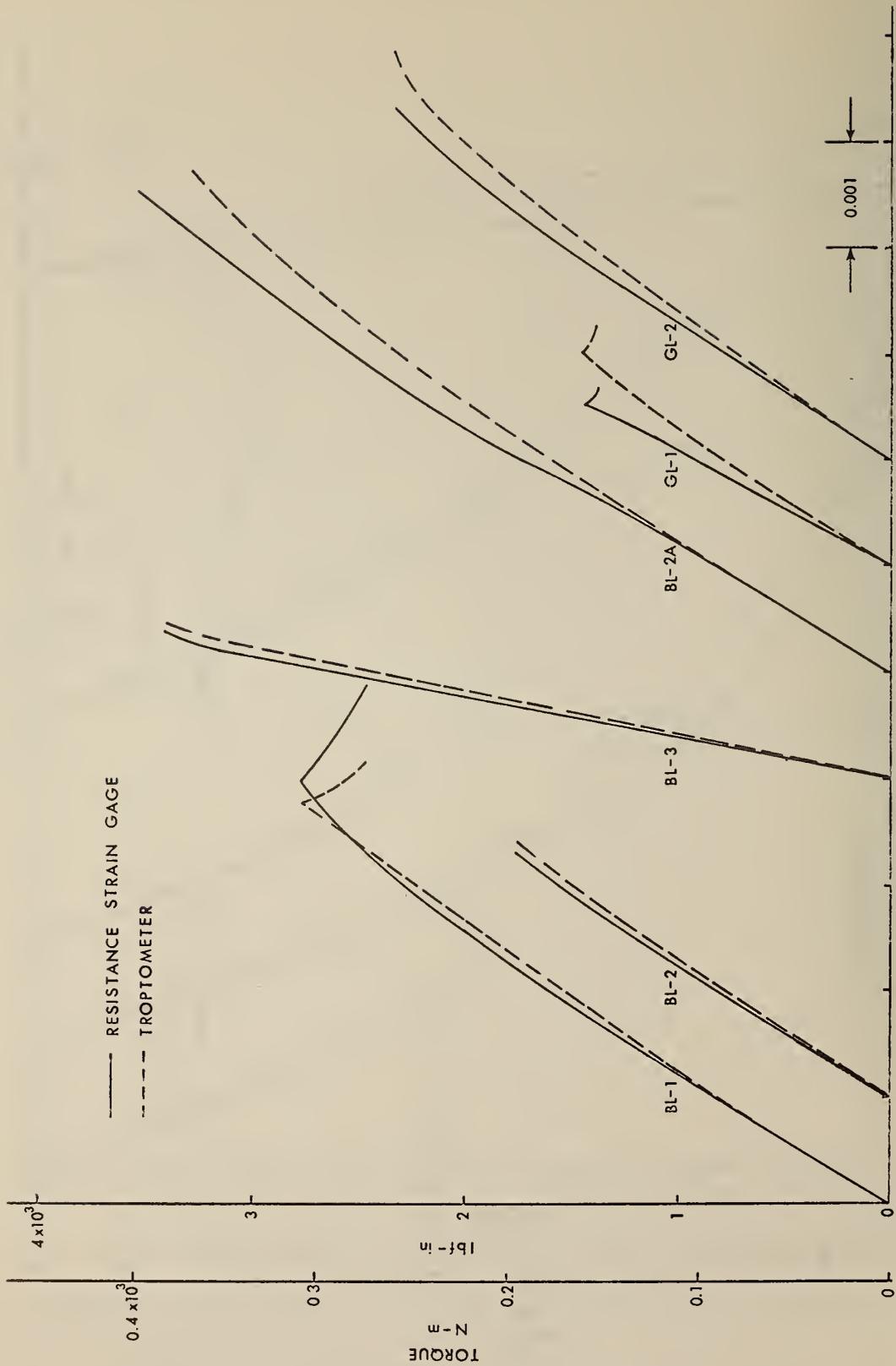


Figure 20h. Torque-shear strain curves for specimens BL-1, BL-2, BL-3, BL-2A, GL-1 and GL-2.

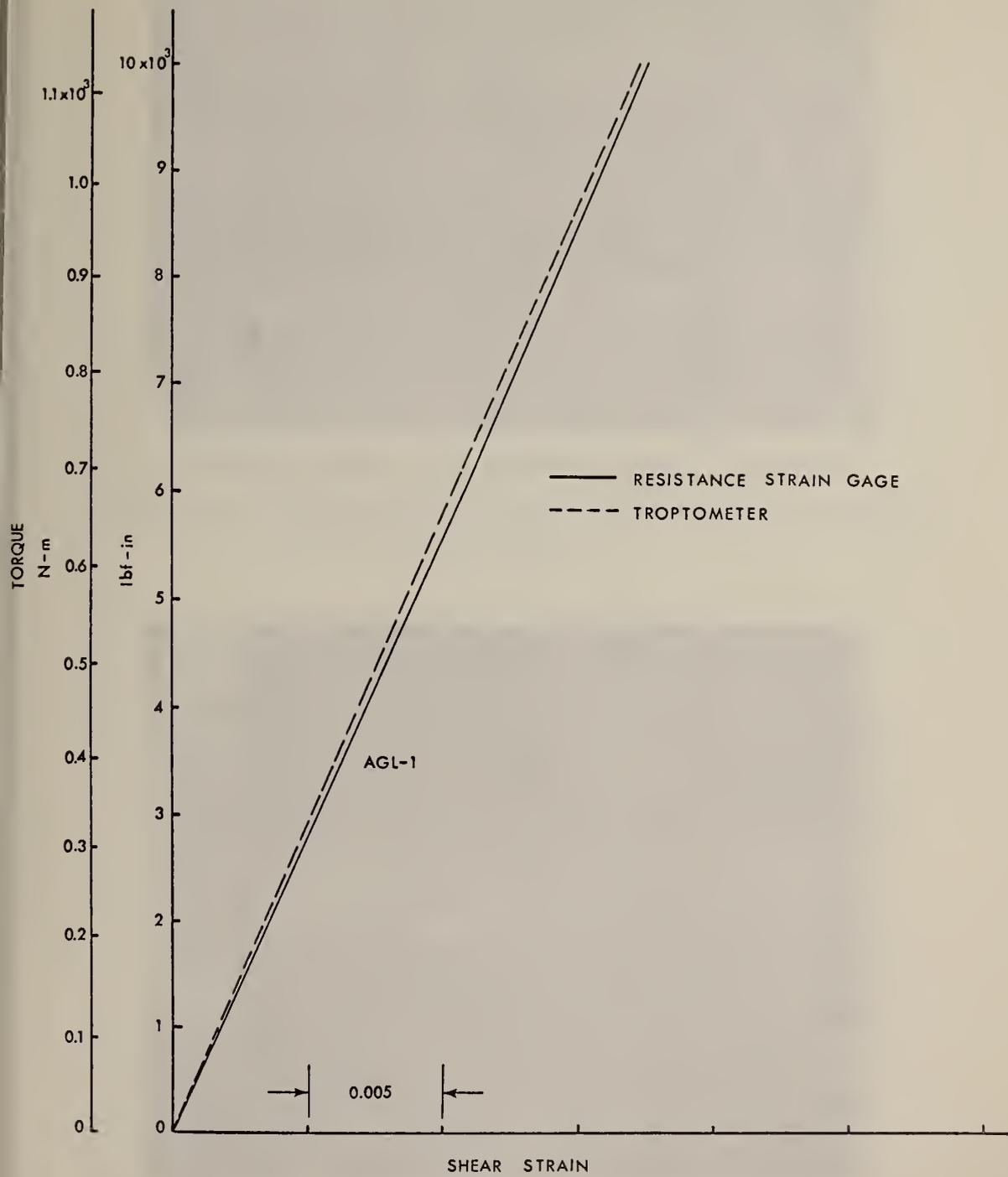


Figure 201. Torque-shear strain curve for specimen AGL-1.

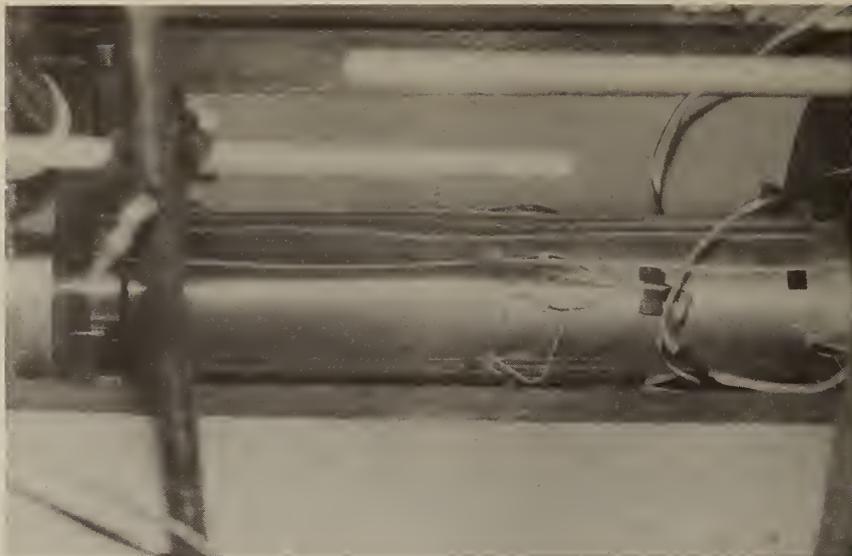


Figure 21a. Typical failure of AL series specimen.



Figure 21b. Typical failure of AS series specimen.



Figure 21c. Buckle failure of ABL series specimen reinforced at  $0^\circ$  degree ply angle.



Figure 21d. Buckle failure of ABL series specimen reinforced at  $90^\circ$  degree ply angle.



Figure 21e. Buckle failure of ABL series specimen reinforced at 45 degree ply angle.

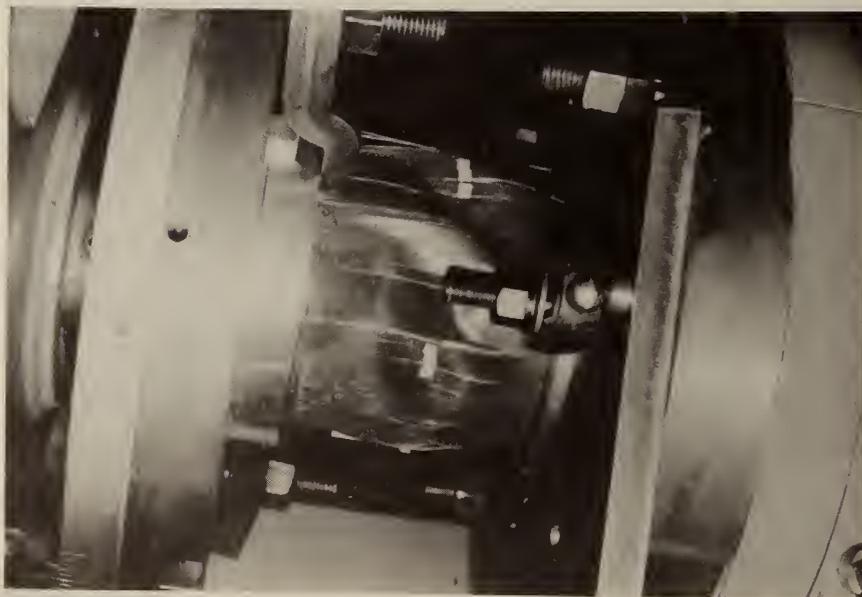


Figure 21f. Buckle failure of ABS series specimen reinforced at a 0 degree ply angle.



Figure 2lg. Buckle failure of an ABS series specimen reinforced at a 90 degree ply angle.

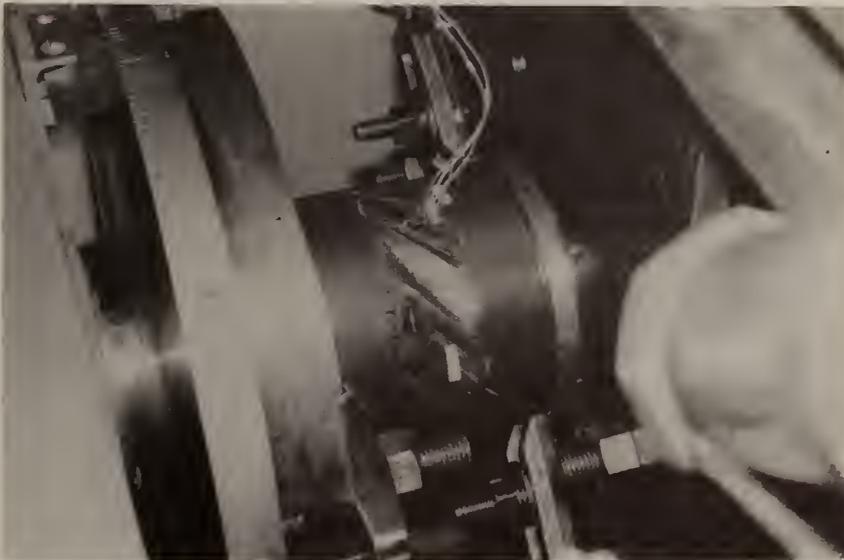


Figure 2lh. Buckle failure of an ABS series specimen reinforced at a 45 degree ply angle.



Figure 21i. Buckle failure of BL series specimen with all plies oriented at 0 degree.



Figure 21j. Material failure of BL series with all plies oriented at 90 degrees.

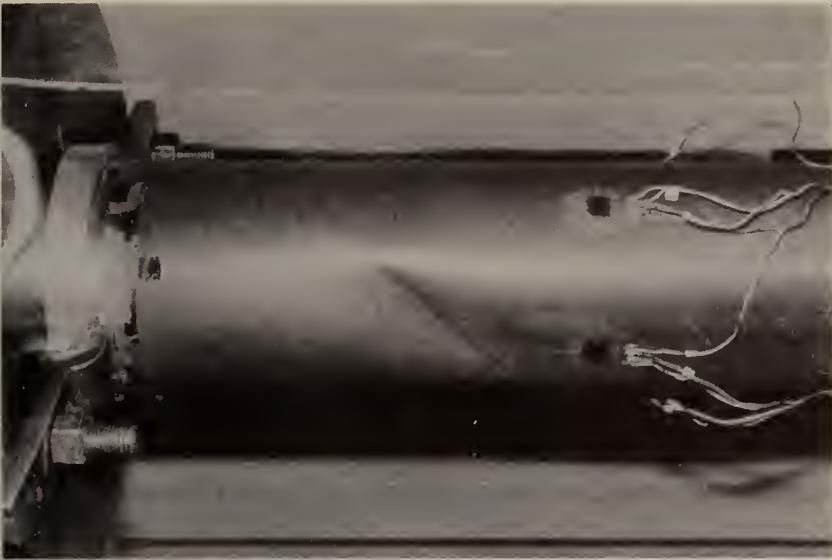


Figure 21k. Material failure which occurred after buckling in a BL series specimen with all plies oriented at +45 degrees.

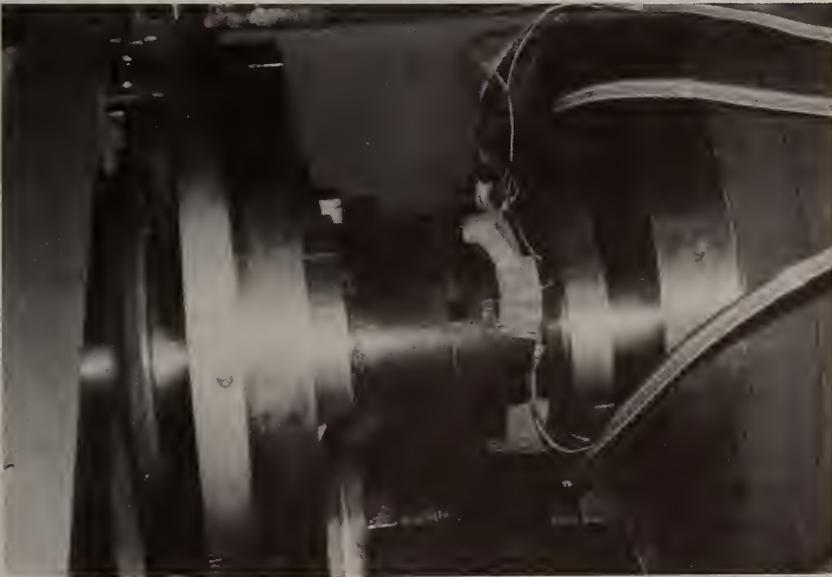


Figure 21l. Material failure in a BS series specimen with all plies oriented at 90 degrees.



Figure 21m. Buckle failure in AGL series specimen reinforced at a ply angle of 0 degrees.

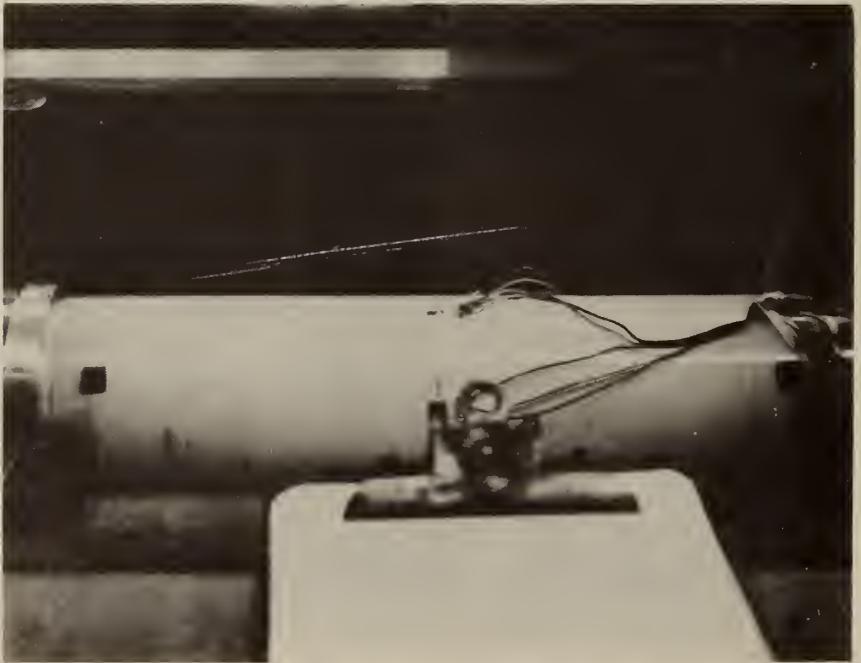


Figure 21n. Buckle failure in GL series specimen with ply angle of 90 degrees.

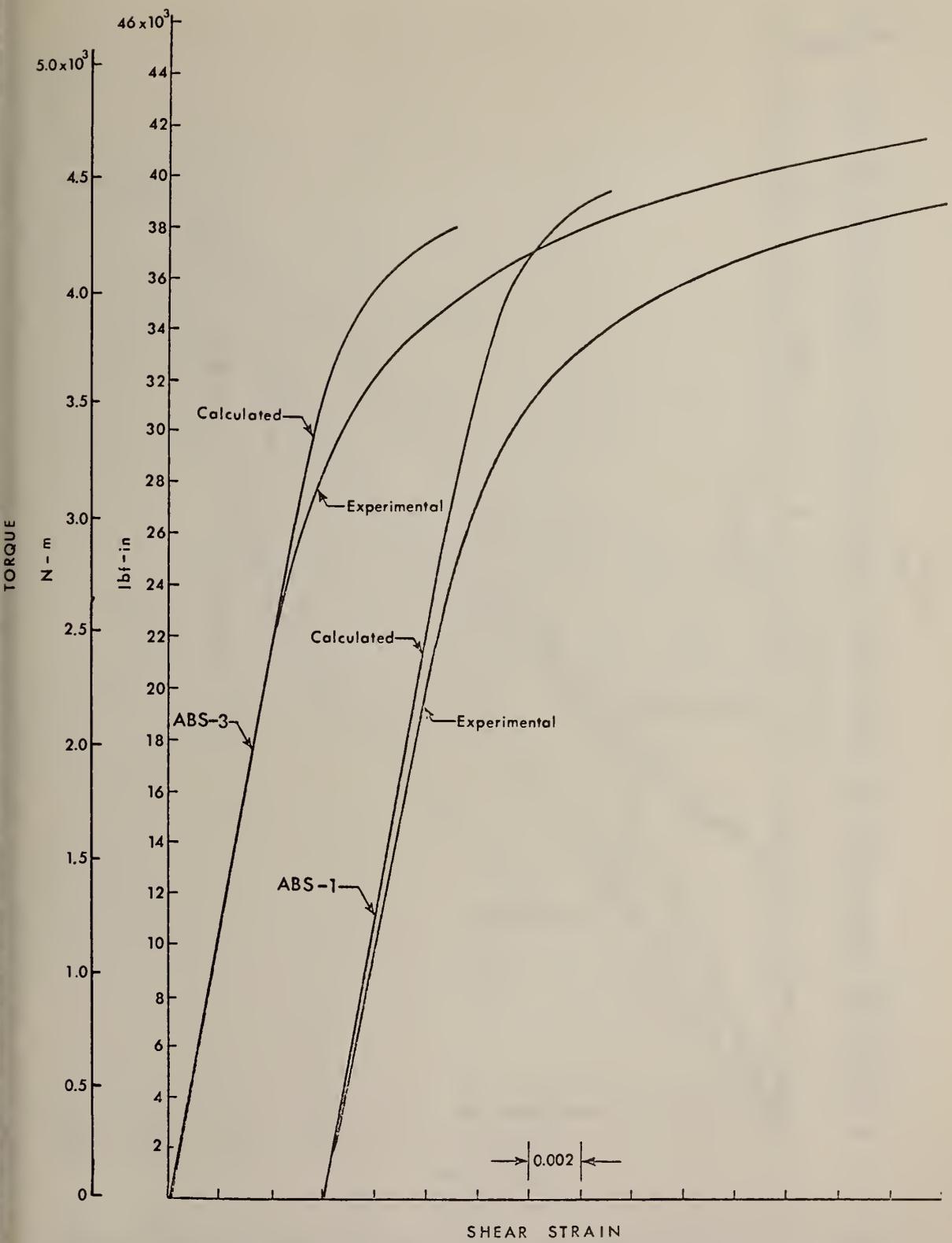


Figure 22a. Experimental and calculated torque-shear strain behavior for ABS specimens with 0-degree reinforcement.

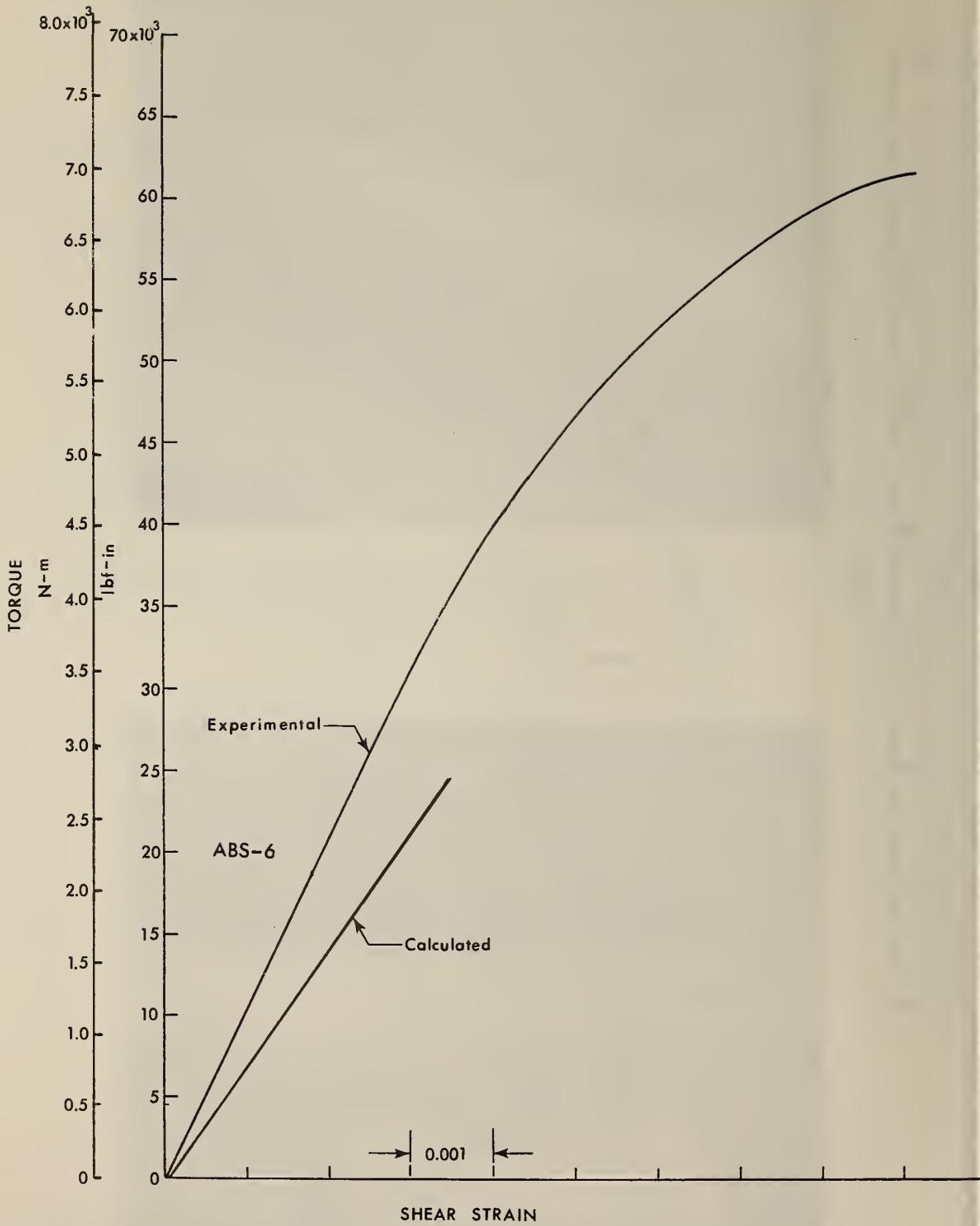


Figure 22b. Experimental and calculated torque-shear strain behavior for ABS specimens with 45-degree reinforcement.

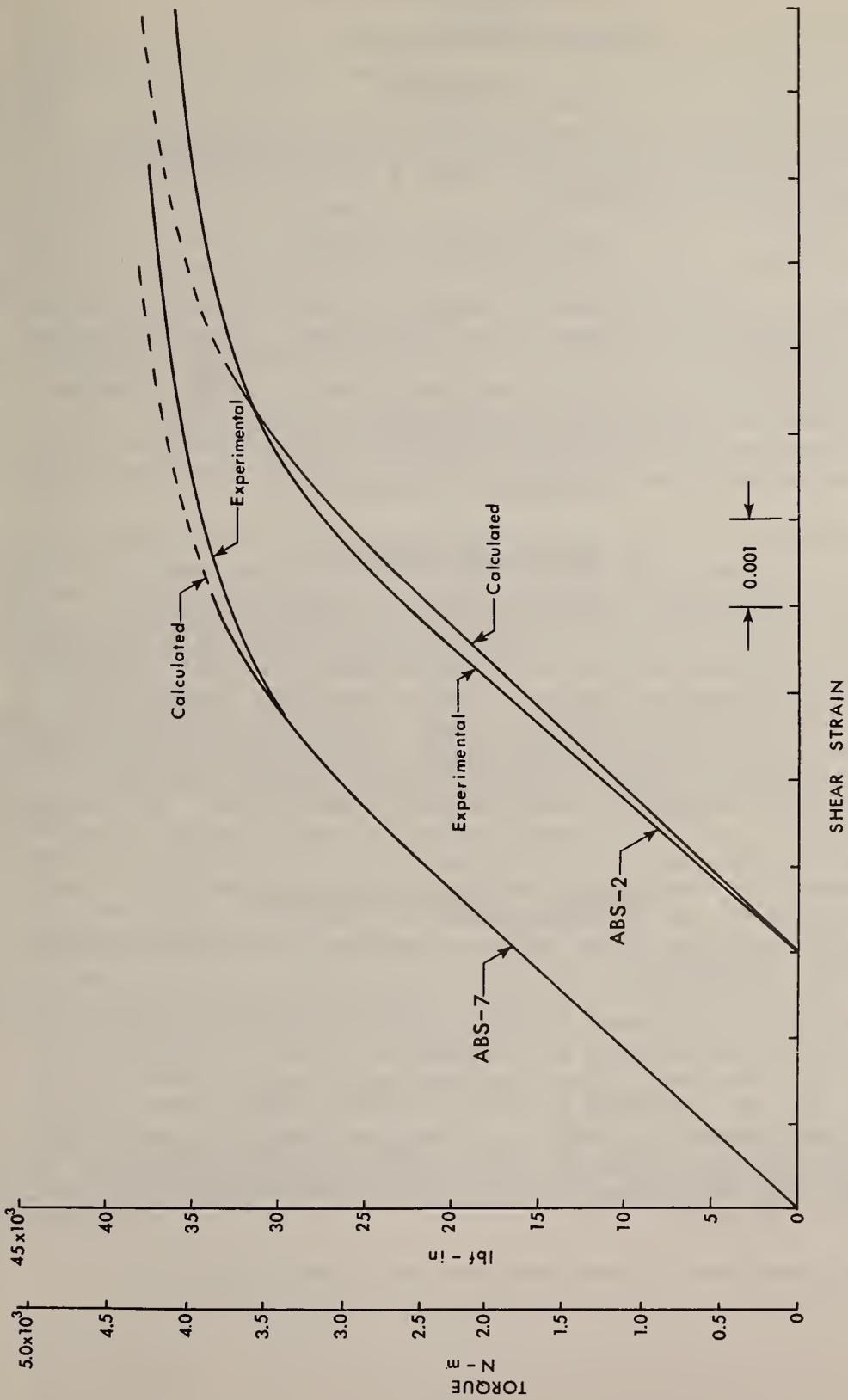


Figure 22c. Experimental and calculated torque-shear strain behavior for ABS specimens with 90-degree reinforcement.

APPENDIX A  
MATERIAL PROPERTY TESTS

A1. SPECIMENS

Material property tests for determination of longitudinal modulus, transverse modulus, and Poisson's ratio were made on all materials used in this program.

a. Aluminum-Alloy Specimens

Tensile specimens as shown in figure A-1 were machined from 6061-T6 rolled aluminum-alloy sheet in two thicknesses. These sheets were 0.020 in (0.05 cm) and 0.032 in (0.08 cm) thick. Twelve specimens were removed from each sheet, three each in the 0, +45, -45 and 90-degree directions with respect to the direction of rolling. In addition, two specimens also shown in figure A-1 were removed from the wall of an untested long-tube specimen in the axial direction of the tube.

b. Boron/Epoxy Specimens

Boron/epoxy tensile coupons as shown in figure A-2 were fabricated from four plies of unidirectional boron/epoxy broadgoods material. The laminates for both 0 and 90-degree material properties were fabricated simultaneously. A dam was machined from a single bakelite sheet 0.032 in (0.08 cm) thick. A shim was placed in the dam to achieve a final laminate thickness of approximately 0.02 in (0.05 cm). The parts were cured in a heated platen press using the manufacturer's recommended time-temperature cure cycle. The laminates were allowed to cool in the press under a minimal pressure after cure. The cured laminates were 3 in (7.6 cm) wide and 12 in (30.5 cm) long. Each laminate was then sliced lengthwise, using a diamond grit saw, into specimens of the dimensions shown in figure A-2. Only the four coupons in the center of the laminate were used for determination of the material properties.

End tabs were prepared for the tensile specimens by curing four plies of fiberglass prepreg into laminates 0.5 in (1.3 cm) wide and 12.0 in (30.5 cm) long. These laminates were cut into tabs 3.0 in (7.7 cm) long and chamfered in a belt sander. Pairs of end tabs were bonded simultaneously to each end of each specimen using epoxy resin. This epoxy was cured using the recommended time-temperature cure cycle. The cured epoxy was allowed to cool to room temperature before the pressure was removed.

The dimensions of these specimens are given in Table A-1.

### c. Fiberglass/Epoxy Specimens

Fiberglass/epoxy tensile coupons as shown in figure A-2 were fabricated from six plies of unidirectional 3-in (7.7-cm) wide tape material. The material was sliced into 0.5-in (1.3-cm) wide, 12-in (30.5-cm) long strips and laid up using techniques similar to those used for boron/epoxy specimens. Only 0-degree-ply-orientation material properties were measured. The coupons were cured in a heated platen press using the recommended time-temperature cure cycle. The laminates were allowed to cool in the press after cure. End tabs, prepared in the same way as those for the boron/epoxy specimens, were bonded to the coupons using the technique described above.

The dimensions of these specimens are also given in table A-1.

## A2. TENSILE TEST PROCEDURE AND RESULTS

### a. Aluminum-Alloy Specimens

The average results from tensile tests on specimens machined from aluminum-alloy sheet material are shown in table A-2. Axial extensions were measured using a linear variable differential transformer (LVDT). The results do not show any significant anisotropic behavior due to sheet rolling.

The specimens machined from a spare tube were instrumented with electrical resistance foil strain gages to measure both the longitudinal and the transverse strains in the specimens. The material properties computed from these results are also shown in table A-2.

### b. Boron/Epoxy Tensile Specimens

These specimens were instrumented with electrical resistance foil strain gages to measure longitudinal and transverse strains. The results from these tests are shown in Table A-3. These specimens failed in the gaged section of the coupon.

### c. Glass/Epoxy Tensile Specimens

These specimens were instrumented with an LVDT to measure axial strains. The results from these tests are shown in table A-3. These specimens always failed in the tab section of the coupon, not in the gaged section.

Table A1 - Dimensions of Boron/Epoxy and Glass/Epoxy Tensile Specimens

Specimen type	Average thickness		Average width		Crossectional area		Orientation deg
	in	cm	in	cm	in <sup>2</sup>	cm <sup>2</sup>	
Boron/epoxy							
TBL-1	0.026	0.066	0.507	1.288	0.013	0.084	0
TBL-2	.025	.064	.510	1.295	.013	.084	0
TBL-3	.023	.058	.507	1.288	.012	.077	0
TBL-4	.023	.058	.507	1.288	.012	.077	0
TBT-1	.026	.066	.509	1.293	.013	.084	90
TBT-2	.027	.069	.514	1.306	.014	.090	90
TBT-3	.024	.061	.512	1.300	.012	.077	90
TBT-4	.024	.061	.516	1.311	.012	.077	90
Glass/epoxy							
1	0.060	0.152	0.756	1.910	0.045	0.290	0
2	.059	.150	.732	1.859	.043	.278	0
3	.059	.150	.732	1.859	.043	.278	0
4	.059	.150	.736	1.869	.043	.278	0
5	.060	.152	.734	1.864	.044	.284	0

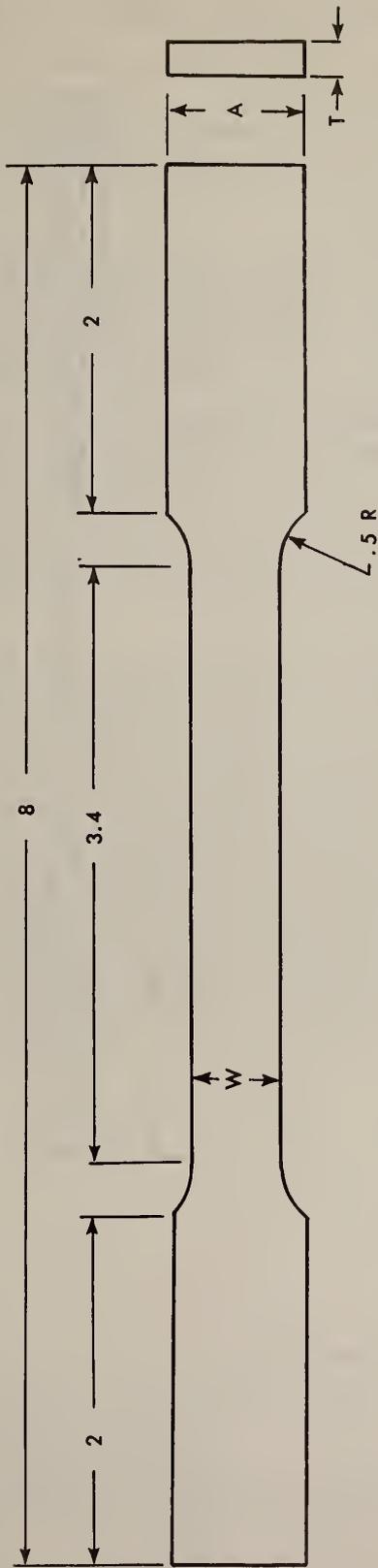
Table A2 - Results of Material Property Tests on Aluminum Alloy Material

Specimen orientation degrees	Specimen thickness		Yield stress		Ultimate breaking stress		Tensile modulus		Poisson's ratio
	in	cm	lbf/in <sup>2</sup>	N/m <sup>2</sup>	lbf/in <sup>2</sup>	N/m <sup>2</sup>	lbf/in <sup>2</sup>	N/m <sup>2</sup>	
Specimens Cut From Rolled Sheet									
0	0.020	0.05	39,500	272 x 10 <sup>6</sup>	43,400	299 x 10 <sup>6</sup>	9.8 x 10 <sup>6</sup>	68 x 10 <sup>9</sup>	
+45	0.020	0.05	38,000	262	42,600	294	9.8	68	
-45	0.020	0.05	37,600	259	42,000	289	9.7	67	
90	0.020	0.05	38,200	263	43,100	297	9.8	68	
0	0.032	0.08	43,500	300	45,100	311	9.8	68	
+45	0.032	0.08	40,300	278	45,100	311	10.2	70	
-45	0.032	0.08	40,600	280	45,000	310	10.2	70	
90	0.032	0.08	42,400	292	45,300	312	9.9	68	
Specimens Cut From Untested Tube									
0	0.023	0.06	44,600	308 x 10 <sup>6</sup>	47,300	326 x 10 <sup>6</sup>	10.2 x 10 <sup>6</sup>	70 x 10 <sup>9</sup>	0.33

Table A3 - Results of Material Property Tests on Composite Material

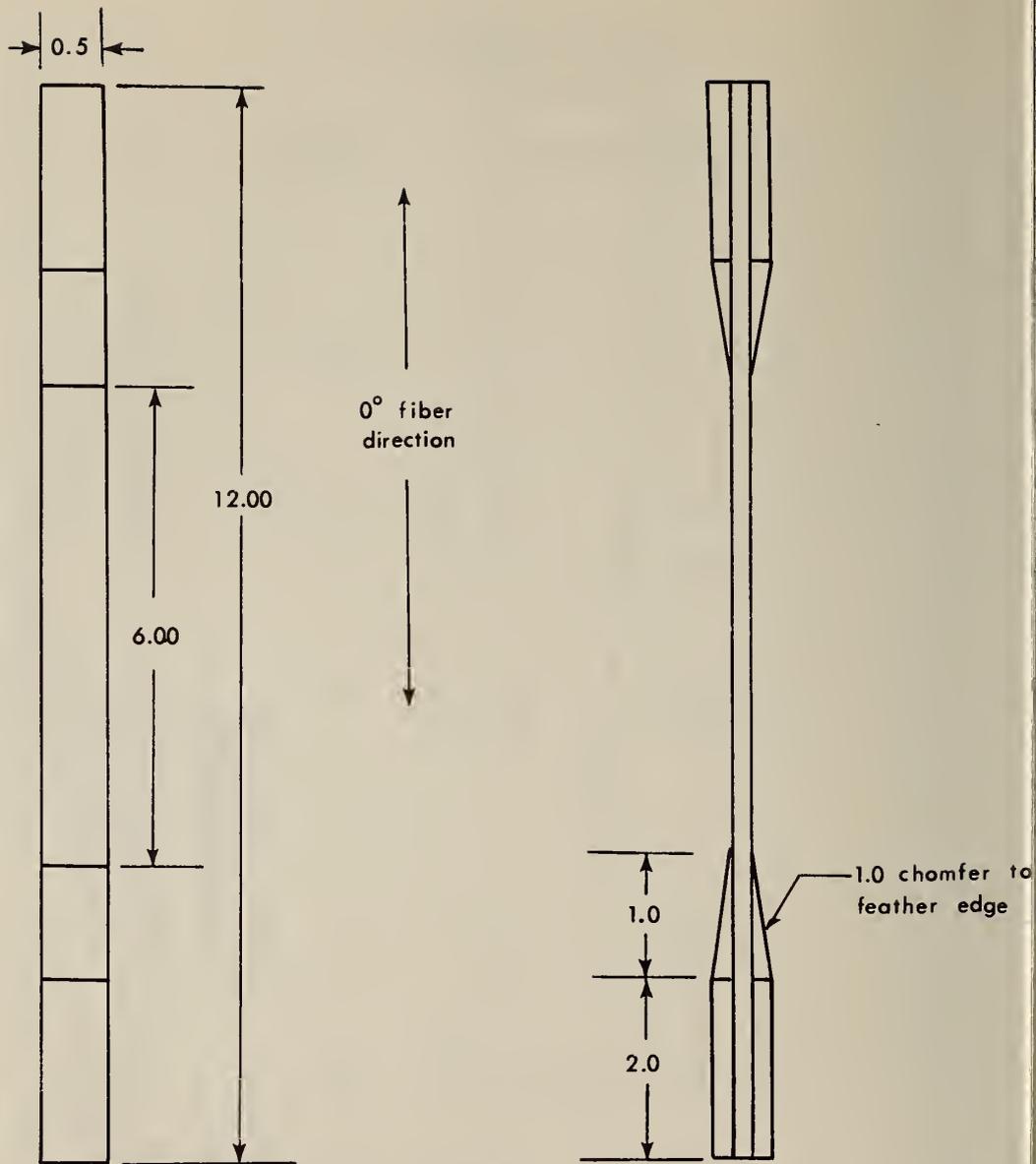
Specimen No.	Fiber volume percent	Ultimate breaking stress		0-degree tension modulus, E <sub>11</sub>		90-degree tension modulus, E <sub>22</sub>		ν <sub>21</sub>	ν <sub>12</sub>
		lbf/in <sup>2</sup>	N/m <sup>2</sup>	lbf/in <sup>2</sup>	N/m <sup>2</sup>	lbf/in <sup>2</sup>	N/m <sup>2</sup>		
Boron/epoxy									
TBT-1	40	5,000	35 x 10 <sup>9</sup>	-	-	2.9 x 10 <sup>6</sup>	20 x 10 <sup>9</sup>	0.02	-
TBT-2	39	4,300	30	-	-	2.4	17	.01	-
TBT-3	44	5,400	38	-	-	2.3	16	(a)	-
TBT-4	44	5,900	41	-	-	2.3	16	(a)	-
Glass/epoxy									
TBL-1	40	142,000	994	29 x 10 <sup>6</sup>	210 x 10 <sup>9</sup>	-	-	-	0.21
TBL-2	42	146,000	1,020	31	220	-	-	-	.23
TBL-3	46	165,000	1,150	31	220	-	-	-	.20
TBL-4	46	162,000	1,130	31	220	-	-	-	.20
1	(a)	121,000	844 x 10 <sup>6</sup>	4.8 x 10 <sup>6</sup>	34 x 10 <sup>9</sup>	-	-	-	-
2	(a)	114,000	794	5.2	36	-	-	-	-
3	(a)	111,000	777	4.9	34	-	-	-	-
4	(a)	121,000	846	4.9	34	-	-	-	-
5	(a)	121,000	845	5.0	35	-	-	-	-

(a) Measurement not made.



SPECIMEN TYPE	W, in	T, in	A, in
SHEET MATERIAL	0.500	0.020	0.750
SHEET MATERIAL	.500	.032	.750
CURVED TUBE MATERIAL	.750	.022	1.00

Figure A-1. Aluminum tensile specimens. All dimensions are given in inches (1 in = 2.54 cm).



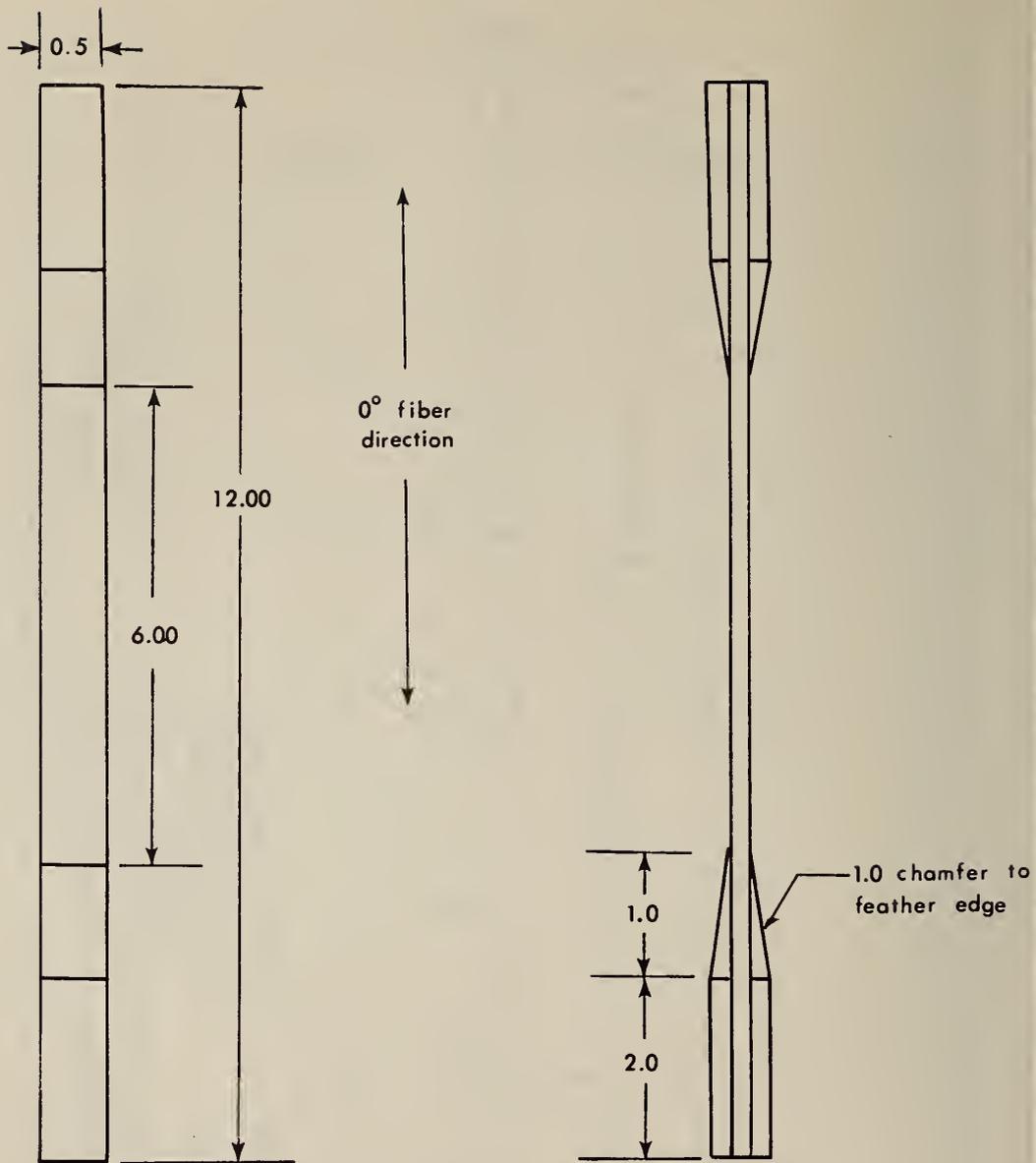
**Notes**

Top ply orientation is 0, 0, 0, 0,

Specimen thickness does not vary by more than 0.002 over its length or width

Figure A-2. Boron/epoxy and glass/epoxy material property specimen. All dimensions are given in inches (1 in = 2.54 cm).

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17. KEY WORDS (Alphabetical order, separated by semicolons) Aircraft structures; boron/epoxy; composite materials; reinforced aluminum; stability; thin shells; torsion buckling.			
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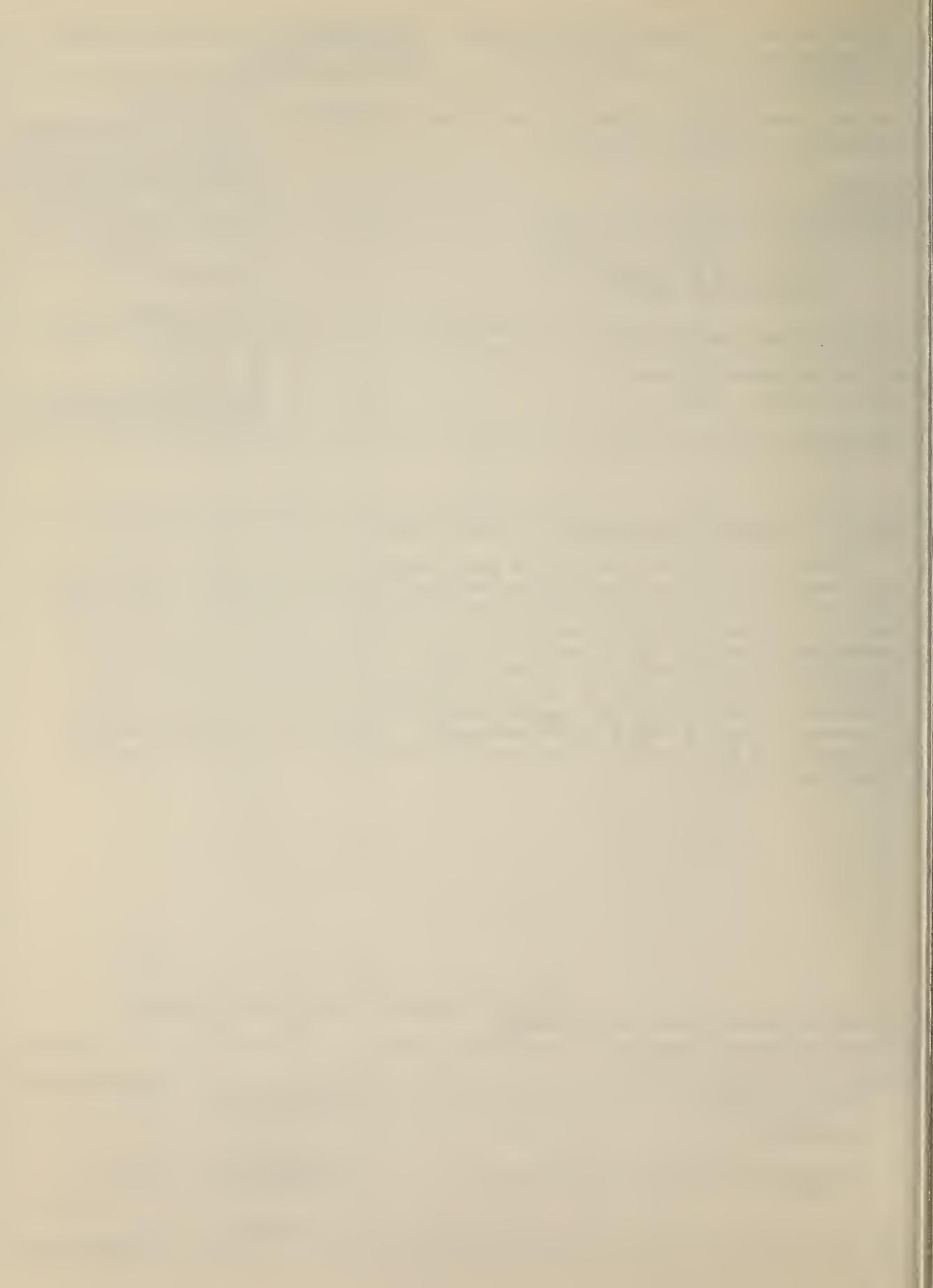
**Notes**

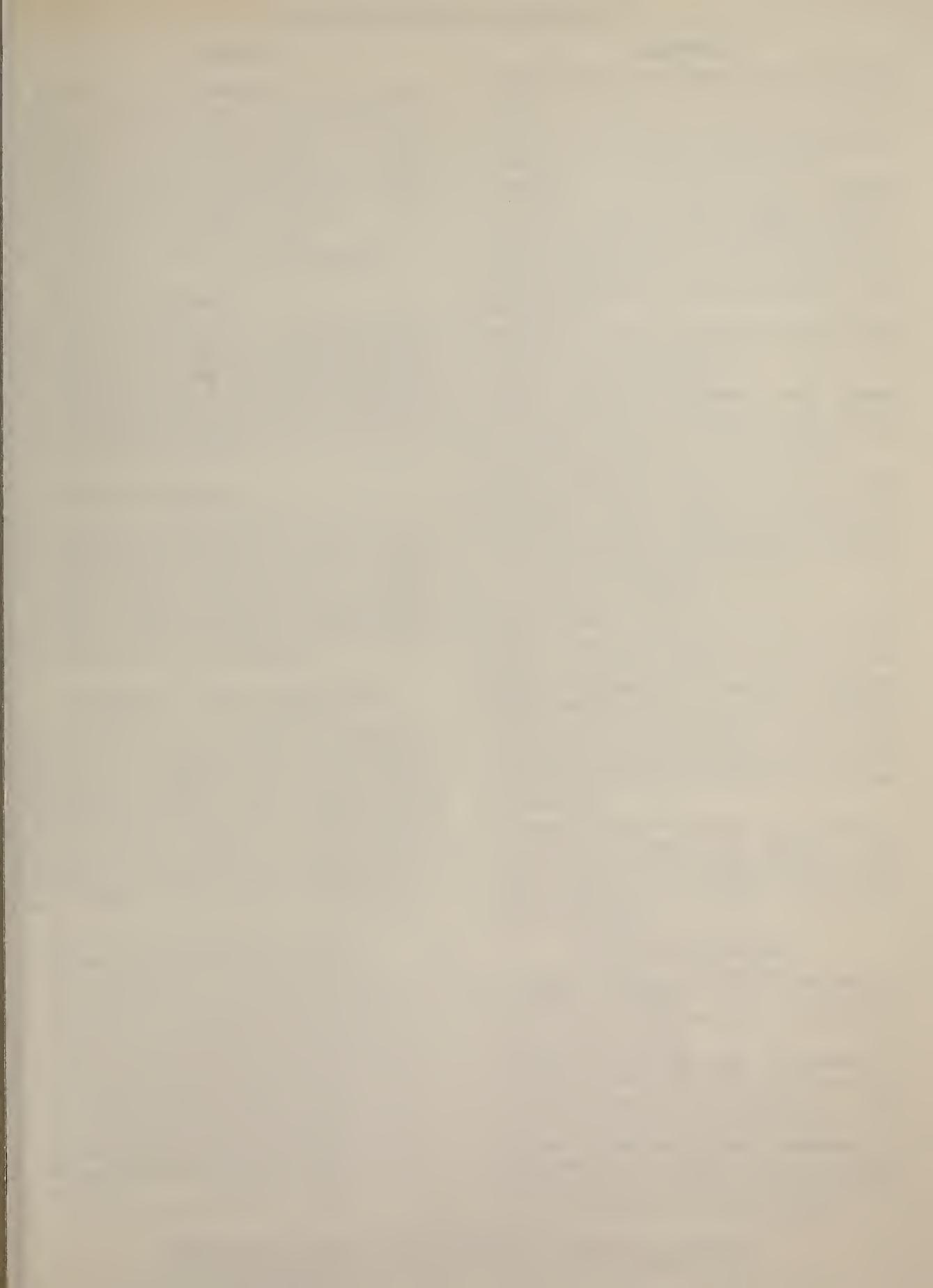
Tab ply orientation is 0, 0, 0, 0,

Specimen thickness does not vary by more than 0.002 over its length or width

Figure A-2. Boron/epoxy and glass/epoxy material property specimen. All dimensions are given in inches (1 in = 2.54 cm).

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